Multiple Choice

1.(5 pts.) Let f, g and h be any three functions such that $f(2) = 3$, $g(2) = 5$, $h(2) = 1$; $f(3) = 3$, $g(3) = 2$, $h(3) = 5$; $f(5) = 2$, $g(5) = 1$, $h(5) = 3$ and $f(1) = 5$, $g(1) = 3$, $h(1) = 2$. If

$$F(x) = (fg)(x) + (f \circ h)(x + 1)$$

what is $F(2)$?

(a) 18 (b) 16 (c) 17 (d) 9

(e) Can not be determined from the given information.

2.(5 pts.) Compute the left handed limit $\lim_{u \to 1^-} \frac{u^2 - 1}{u^2 + 1}$.

(a) 0 (b) $-\infty$ (c) ∞ (d) 1

(e) Does not exist and is not ∞ or $-\infty$.

3.(5 pts.) Compute the right handed limit $\lim_{y \to \frac{\pi}{2}^+} \tan y$.

(a) 0 (b) $-\infty$ (c) ∞ (d) 1

(e) Does not exist and is neither ∞ nor $-\infty$.

4.(5 pts.) The function $f(x) = \frac{x^2 - 1}{x^3 - 4x}$ is continuous everywhere *except* at

(a) $x = \pm 2$ (b) $x = 0$ and $x = \pm 1$

(c) $x = 0$, $x = \pm 1$ and $x = \pm 2$ (d) $x = 0$ and $x = \pm 2$

(e) f is a rational function and so it is continuous everywhere.

5.(5 pts.) If $f(x) = (x^2 + 3x)(6x^5 - 2x^8)$ compute $f'(1)$.

(a) 76 (b) 70 (c) -36 (d) 16 (e) 67

6.(5 pts.) If $f(x) = \sqrt[3]{x^5} + \dfrac{6}{\sqrt[5]{x^3}}$, then $f'(x) =$?

(a) $\dfrac{5\sqrt[3]{x^2}}{3} + \dfrac{5}{18\sqrt[5]{x^8}}$ (b) $\dfrac{3\sqrt[3]{x^2}}{5} - \dfrac{5}{18\sqrt[5]{x^8}}$ (c) $\dfrac{3\sqrt[3]{x^2}}{5} - \dfrac{18}{5\sqrt[5]{x^8}}$

(d) $\dfrac{3\sqrt[3]{x^2}}{5} + \dfrac{18}{5\sqrt[5]{x^8}}$ (e) $\dfrac{5\sqrt[3]{x^2}}{3} - \dfrac{18}{5\sqrt[5]{x^8}}$

7.(5 pts.) If $f(x) = \dfrac{x + \cos x}{x + \sin x}$ compute $f'(x)$.

(a) $\dfrac{(1 - \sin x)(x + \sin x) - (x + \cos x)(1 + \cos x)}{(x + \cos x)^2}$

(b) $\dfrac{(1 - \sin x)(x + \sin x) - (x + \cos x)(1 + \cos x)}{(x + \sin x)^2}$

(c) $\dfrac{(1 - \cos x)(x + \sin x) - (x + \cos x)(1 + \sin x)}{(x + \cos x)^2}$

(d) $\dfrac{(1 - \cos x)(x + \sin x) - (x + \cos x)(1 + \sin x)}{(x + \sin x)^2}$

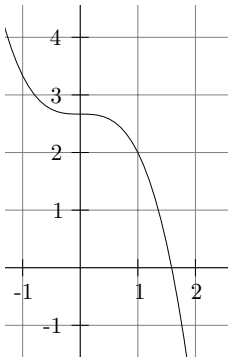
(e) $-\csc^2 x$

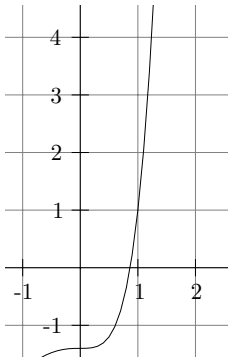
8.(5 pts.) In preparation for Halloween, find all the horizontal tangent lines to the witch of Maria Agnesi. The witch of Maria Agnesi is the graph of $y = \dfrac{1}{1 + x^2}$.

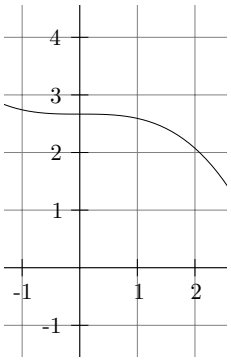
(a) $y = \pm\dfrac{1}{2}$ (b) $y = \dfrac{1}{2}$ (c) $y = 1$

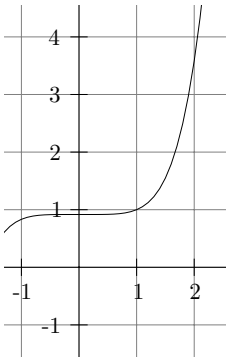
(d) $y = \dfrac{1 - x^2}{(1 + x^2)^2}$ (e) $y = \dfrac{1}{3}$

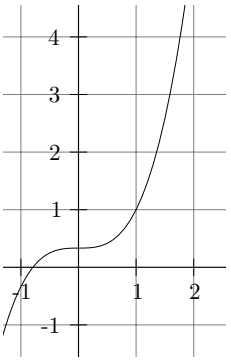
9.(5 pts.) For which graph below is the slope of the tangent line at $\big(1, f(1)\big)$ equal to 2?

(a) 

(b) 

(c) 

(d) 

(e) 

10.(5 pts.) What is $\lim\limits_{y \to \frac{\pi}{4}} \dfrac{(\tan y) - 1}{y - \frac{\pi}{4}}$?

(a) sec(2) (b) Does not exist. (c) 1 (d) 2

(e) $\dfrac{1}{2}$

Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(10 pts.) The limit $\lim_{x\to 0} \cos\left(\frac{1}{x}\right)$ does not exist, but the limit $\lim_{x\to 0} x\cos\left(\frac{1}{x}\right) = 0$. It follows that the function

$$f(x) = \begin{cases} x\cos\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

is continuous. It also follows easily that the function

$$g(x) = \begin{cases} x^2\cos\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

is continuous.

a) Using the definition of the derivative, show f is not differentiable at $x = 0$.

b) Using the definition of the derivative, show g is differentiable at $x = 0$ and compute the value of $g'(0)$.

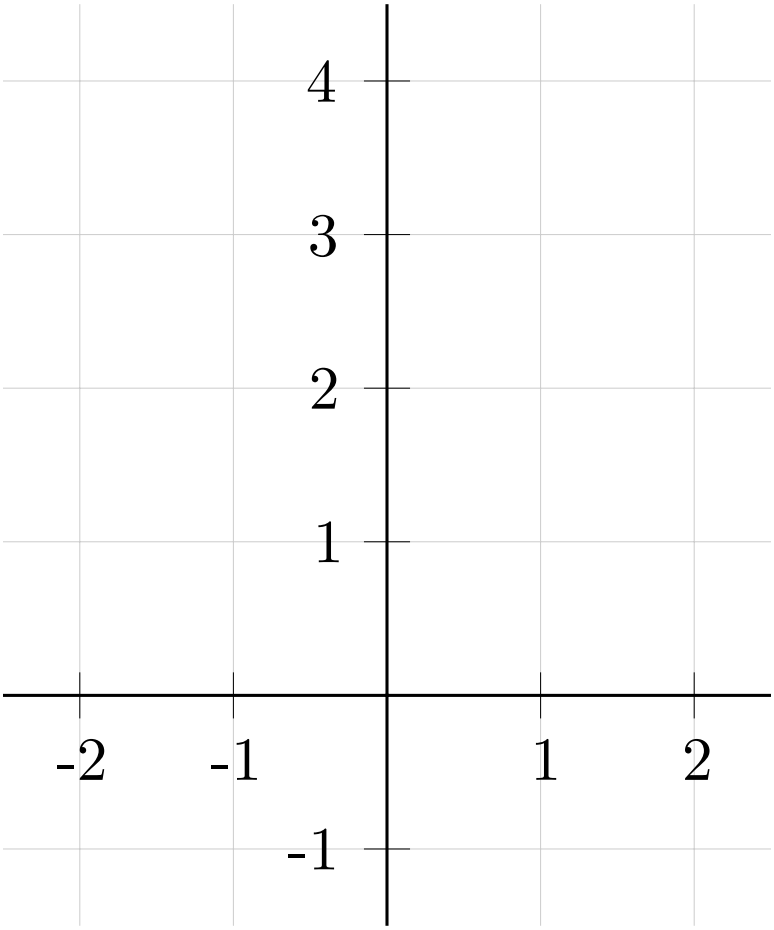
12.(10 pts.) At what point(s) on the graph of the function $y = x^2 - 2x + 4$ does the tangent line at that point pass through the origin?
Hint: Write down the equation for the tangent line through the point $(a, a^2 - 2a + 4)$ and proceed from there.

13.(10 pts.) Show that the equation

$$\frac{\sin x}{x} = x$$

has at least one solution. Be sure to check the hypotheses of any theorem you might use.

14.(10 pts.) Draw a graph for a continuous function $y = f(x)$ which satisfies all the conditions $f(1) = 0$, $f'(1) = -1$, $f(0) = 1$, $f'(0) = 1$ and $f(-1) = 3$.



15.(10 pts.) A missile is launched straight up with the engines firing in such a way that the height above the ground at all times is given by $s(t) = 12t - t^3$ where t is measured in minutes and s is measured in miles.

a) How high does the missile get?

b) What is the impact velocity? (The impact velocity is the instantaneous velocity the missile has as it hits the ground. This is not a trick question – the answer is not 0.)

Name: ANSWERS

Instructor: ANSWERS

Exam I
September 25, 2003

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for one hour.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 6 pages of the test.

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(•)	(d)	(e)
2.	(•)	(b)	(c)	(d)	(e)
3.	(a)	(•)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(•)	(e)
5.	(•)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(•)
7.	(a)	(•)	(c)	(d)	(e)
8.	(a)	(b)	(•)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(•)
10.	(a)	(b)	(c)	(•)	(e)

DO NOT WRITE IN THIS BOX!

Total multiple choice: _____

11. _____

12. _____

13. _____

14. _____

15. _____

Total: _____