Multiple Choice

1.(5 pts.) A car is traveling along a straight highway. The position at time t is given by $s(t) = -t^3 + 10t^2 + 7t$. At exactly one of the five times below the car is slowing down. Which time is it?

(a) 4   (b) 2   (c) 3   (d) 1   (e) 0

2.(5 pts.) Find $\dfrac{d^3}{dx^3}\left(x^5 + 2x^4 - 7x^3 + 3x^2 + 9x - 8\right)$

(a) $60x^3 - 48x^2 + 8x$ (b) $60x^2 + 48x - 42$ (c) $60x^3 + 6$

(d) $60x^2 + 24x - 6$ (e) $60x^2 - 42x + 48$

3.(5 pts.) Find the differential of the function $f(x) = 2x^3 - 4x^2 + 7x + 10$ when $x = 1$ and $dx = 0.1$.

(a) 0.25   (b) 0.75   (c) 1   (d) 0.5   (e) 0

4.(5 pts.) Consider the function $f(x) = x^3 - 3x + 2003$. The critical numbers of f are -1 and 1. The function f has

(a)   an absolute minimum at $x = -1$ and a local maximum at $x = 1$.

(b)   a local minimum at $x = -1$ and an absolute maximum at $x = 1$.

(c)   an absolute minimum at $x = -1$ and an absolute maximum at $x = 1$.

(d)   a local maximum at $x = -1$ and a local minimum at $x = 1$.

(e)   an absolute maximum at $x = -1$ and a local minimum at $x = 1$.

5.(5 pts.) On which one of the given intervals is the function $f(x) = \sin(2x)$ concave downwards?

(a) $\left(0, \frac{\pi}{2}\right)$ (b) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (c) $\left(\frac{\pi}{2}, \pi\right)$ (d) $\left(0, \pi\right)$ (e) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

2

6.(5 pts.) Find the following derivative:

$$\frac{d}{dx}\left(\sin^{2003}\left(\sqrt{x}\right)\right).$$

(a) $2003\sin^{2002}\left(\dfrac{1}{2\sqrt{x}}\right)$

(b) $2003\sin^{2002}\left(\sqrt{x}\right)\cdot\cos\left(\sqrt{x}\right)\cdot\dfrac{1}{2\sqrt{x}}$

(c) $2003\sin^{2002}\left(\cos\left(\dfrac{1}{2\sqrt{x}}\right)\right)$

(d) $2003\sin^{2002}\left(\sqrt{x}\right)\cdot\cos\left(\dfrac{1}{2\sqrt{x}}\right)$

(e) $2003\sin^{2002}\left(\dfrac{1}{\sqrt{x}}\right)$

7.(5 pts.) Find an equation of the tangent line to the curve $x^2 - 4y^2 = 0$ at the point $(2, 1)$.

(a) $y = \dfrac{1}{4}x + \dfrac{1}{2}$

(b) $y = \dfrac{1}{3}x + \dfrac{2}{3}$

(c) $y = \dfrac{1}{2}x + 1$

(d) $y = \dfrac{1}{2}x$

(e) $y = x - 1$

8.(5 pts.) Find $y'(x)$ for $x^2y^2 - 2x = 4 - 4y$.

(a) $y'(x) = \dfrac{2 - 4xy^2}{2x^2y - 1}$

(b) $y'(x) = \dfrac{2 + xy^2}{2x^2y - 4}$

(c) $y'(x) = \dfrac{2 - 2xy^2}{2x^2y + 4}$

(d) $y'(x) = \dfrac{2 - xy^2}{2x^2y + 1}$

(e) $y'(x) = \dfrac{1 + 2xy^2}{x^2y + 4}$

9.(5 pts.) Find $y''(x)$ for the following function:

$$y = (3x - 1)^5 + (2x + 1)^2 \ .$$

(a) $y'' = 160(3x - 1)^4 + 6$

(b) $y'' = 160(3x - 1)^3 + 8$

(c) $y'' = 100(3x - 1)^4 + 2$

(d) $y'' = 120(3x - 1)^3 + 4$

(e) $y'' = 180(3x - 1)^3 + 8$

3

10.(5 pts.)   Find all critical numbers of

$$f(x) = \frac{2x^2}{x+2}.$$

(a) $x = 0$, $x = -2$ and $x = -4$ (b) $x = 0$, $x = -2$ and $x = -1$

(c) $x = 0$, $x = -2$ and $x = 4$ (d) $x = 1$ and $x = -4$

(e) $x = 0$ and $x = -4$

Partial Credit
You must show your work on the partial credit problems to receive credit!

11.(10 pts.)   On the interval $[0, 1]$ the polynomial $p(x) = x^5 + 2x^3 + 2x - 3$ has at least one root by the Intermediate Value Theorem since a polynomial is continuous on any interval and $p(0) = -3 < 0$ and $p(1) = 1 + 5 + 2 - 3 = 5 > 0$. Show that there is only one root of this polynomial on this interval.

12.(10 pts.)   Consider the function $f(x) = 3x^4 - 4x^3 + 2003$ defined on the real axis.
  a) find the absolute minimum and absolute maximum of f (if they exist);
  b) find the intervals of increase and decrease;
  c) find the intervals of concavity and inflection points.

13.(10 pts.)   Suppose an oil spill has taken the form of a circular region and its area is increasing at the rate of 100 square meters per hour. At what rate the radius of the region increasing when the radius is 200 meters? Justify your answer.

14.(10 pts.)   After t hours, the number of bacteria in a laboratory culture is given by

$$n = 6t^2 + 200 .$$

Use differentials to approximate Δn, the change in the number of bacteria when t changes from 5 hours to 5.2 hours. Justify your answer.

15.(10 pts.)   Find the absolute maximum and minimum values of the following function

$$f(x) = 2 - x^{\frac{2}{3}}$$

on the interval $[-2, 2]$. Justify your answer.

Exam II
October 28, 2003

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for one hour.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 4 pages of the test.

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.   ($\bullet$)   (b)   (c)   (d)   (e)

2.   (a)   ($\bullet$)   (c)   (d)   (e)

3.   (a)   (b)   (c)   ($\bullet$)   (e)

4.   (a)   (b)   (c)   ($\bullet$)   (e)

5.   ($\bullet$)   (b)   (c)   (d)   (e)

6.   (a)   ($\bullet$)   (c)   (d)   (e)

7.   (a)   (b)   (c)   ($\bullet$)   (e)

8.   (a)   (b)   ($\bullet$)   (d)   (e)

9.   (a)   (b)   (c)   (d)   ($\bullet$)

10.   (a)   (b)   (c)   (d)   ($\bullet$)

DO NOT WRITE IN THIS BOX!

Total multiple choice: _____

11.   _____

12.   _____

13.   _____

14.   _____

15.   _____

Total: _____