

Multiple Choice

1.(5 pts.) If

$$f'(x) = \frac{(x-1)^2x}{(x+1)^3},$$

find the local maxima and minima of $f(x)$ assuming that the domain of $f(x)$ is all $x \neq -1$.
Note: you are given f' , not f .

- (a) f has a local minimum at $x = 0$; f has local maxima at $x = 1$ and $x = -1$.
- (b) There are no local minima or local maxima.
- (c) f has a local minimum at $x = 0$; there is no local maximum.
- (d) f has a local minimum at $x = 0$; f has a local maximum at $x = 1$.
- (e) f has a local minimum at $x = 1$; f has a local maximum at $x = -1$.

2.(5 pts.) If

$$f'(x) = \frac{x}{1+x^2}$$

and if $f(x)$ is defined for all x , find the x coordinates of the points of inflection.
Note: you are given f' , not f or f'' .

- (a) Not enough information is given to find the points of inflection.
- (b) $x = 1$.
- (c) $x = 0$.
- (d) $x = -1$ and $x = 1$.
- (e) There are no points of inflection.

3.(5 pts.) If

$$y = \frac{\sqrt[3]{3x^3 + 2x + 3}}{2x + 4},$$

find the horizontal and vertical asymptotes.

- (a) $y = \frac{4}{3}$ is a horizontal asymptote; $x = 2$ is a vertical asymptote.
- (b) There are no horizontal asymptotes; $x = -2$ is a vertical asymptote.
- (c) $y = \frac{3}{2}$ is a horizontal asymptote; $x = -2$ is a vertical asymptote.
- (d) $y = 0$ is a horizontal asymptote; $x = -2$ is a vertical asymptote.
- (e) $y = \frac{\sqrt[3]{3}}{2}$ is a horizontal asymptote; $x = -2$ is a vertical asymptote.

4.(5 pts.) Which formula below is the limit of Riemann sums for $\int_0^1 (1 - x^2)dx$ using the **left-hand-end-point** Riemann sums?

- (a) $\lim_{n \rightarrow \infty} \sum_{i=0}^{n-1} \left(1 - \frac{i^2}{n^2}\right) \cdot \frac{1}{n}$
- (b) $\lim_{n \rightarrow \infty} \sum_{i=0}^n \left(1 - \frac{i^2}{n^2}\right) \cdot \frac{1}{n}$
- (c) $\lim_{n \rightarrow \infty} \sum_{i=0}^{n-2} \left(1 - \frac{i^2}{n^2}\right) \cdot \frac{1}{n}$
- (d) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(1 - \frac{i^2}{n^2}\right) \cdot \frac{1}{n}$
- (e) $\lim_{n \rightarrow \infty} \sum_{i=2}^n \left(1 - \frac{i^2}{n^2}\right) \cdot \frac{1}{n}$

5.(5 pts.) Calculate the following indefinite integral

$$\int \frac{x^2 + 1}{\sqrt{x}} dx =$$

- (a) $\frac{x^3}{\frac{2}{3}x^{\frac{3}{2}}} + x$
- (b) $\frac{2}{5}x^{\frac{5}{2}} + 2x^{\frac{1}{2}} + C$
- (c) $\frac{2}{5}x^{\frac{5}{2}} + 2x^{\frac{1}{2}}$
- (d) $\frac{2}{5}x^{\frac{5}{2}} + \frac{2}{3}x^{\frac{1}{2}}$
- (e) $\frac{2}{5}x^{\frac{5}{2}} + \frac{2}{3}x^{\frac{1}{2}} + C$

6.(5 pts.) Calculate the following definite integral

$$\int_0^3 |x - 1| dx =$$

- (a) $\frac{5}{2}$. (b) -2 . (c) $\frac{9}{2}$. (d) $\frac{3}{2}$. (e) 1 .

7.(5 pts.) Let $g(x) = \int_2^{\sec x} \sqrt{1+t^2} dt$ with $0 \leq x \leq \frac{\pi}{3}$. What is $g'(x)$?

- (a) $(\sqrt{1 + \sec^2 x}) \sec x \tan x$. (b) $g'(x)$ does not exist.
(c) $(\sqrt{1 + x^2}) \sec x \tan x$. (d) $\sqrt{1 + \sec^2 x}$.
(e) $(\sqrt{1 + \sec^2 x}) \tan^2 x$.

8.(5 pts.) What is the indefinite integral

$$\int x\sqrt{x-1} dx = ?$$

- (a) $\frac{x^3}{3} - \frac{x^2}{2} + C$. (b) $\frac{2}{5}x^{\frac{5}{2}} + \frac{2}{3}x^{\frac{3}{2}}$.
(c) $\frac{2}{5}x^{\frac{5}{2}} + \frac{2}{3}x^{\frac{3}{2}} + C$. (d) $\frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}}$.
(e) $\frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}} + C$.

9.(5 pts.) Find the definite integral

$$\int_{-1}^1 x^3(x^2 + 1)^2 dx .$$

- (a) $\frac{17}{12}$. (b) $\frac{17}{24}$. (c) 2 . (d) 0 . (e) 1 .

10.(5 pts.) Let $A = \int_0^1 \frac{x}{x+1} dx$. Which of the following is true of A ?

- (a) $\frac{1}{2} \leq A \leq \frac{3}{4}$. (b) $\frac{-1}{4} \leq A \leq 0$. (c) A is undefined.
- (d) $0 \leq A \leq \frac{1}{2}$. (e) $A = 0$.

Partial Credit

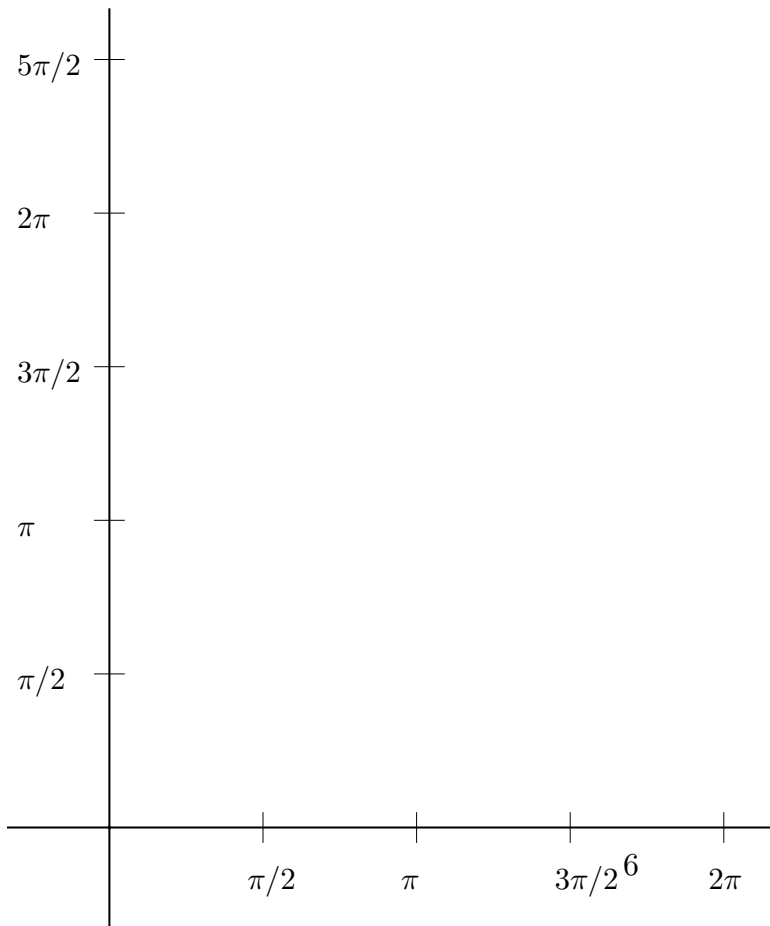
You must show your work on the partial credit problems to receive credit!

11.(10 pts.) Answer the 8 questions, 1, 1a)-1c) and 2, 2a)-2c) below, and use your answers to graph

$$y = x + 2 \cos x$$

on the interval $[0, 2\pi]$? Use the back of the previous page for your calculations.

- 1) $y' =$ _____
 - a) On what interval(s) is y decreasing? _____
 - b) Give both coordinates of any local maxima. _____
 - c) Give both coordinates of any local minima. _____
- 2) $y'' =$ _____
 - a) On what interval(s) is y concave down? _____
 - b) Give both coordinates of any points of inflection. _____
 - c) Give the slope of the tangent line at any points of inflection. _____



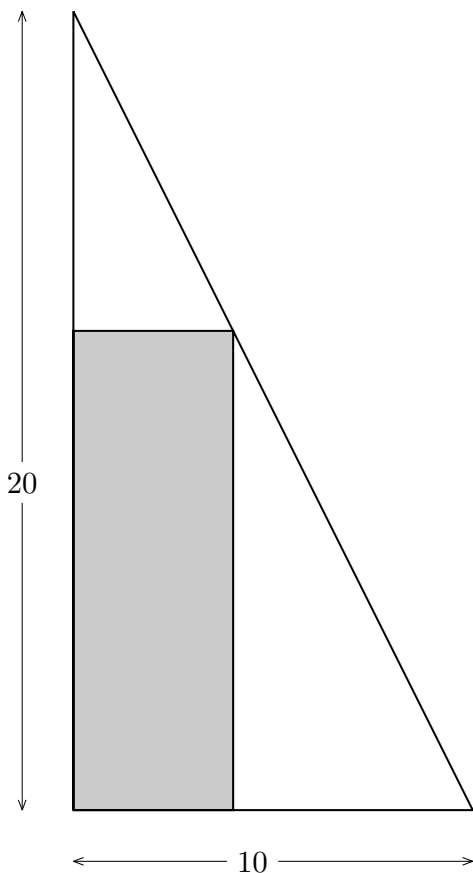
12.(10 pts.) A ball is thrown upward from a height of 256 feet above the ground, with an initial velocity of 96 feet per second. The velocity of the ball at time t is $96 - 32t$ feet per second.

- (a) Find $s(t)$, the function giving the height of the ball at time t .
- (b) How long will the ball take to reach the ground?
- (c) How high will the ball go?

13.(10 pts.) Calculate the area bounded by the curves $y = x^2 + 2x + 3$ and $y = 2x + 4$.

14.(10 pts.) An open rectangular box (that is, a box with no top) with square base and with a volume of 125 cubic feet is needed. Material for the base costs \$6 per square foot, and material for the sides costs \$3 per square foot. **Determine the dimensions** of the box that will minimize the cost of materials. *Justify that your answer is a minimum.*

15.(10 pts.) **Determine the dimensions** of the rectangle of the largest area that can be inscribed in a right triangle with base 10 centimeters and height 20 centimeters. *Justify that your answer is a maximum.*



Name: ANSWERS

Instructor: ANSWERS

Exam III
December 3, 2002

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for one hour.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 7 pages of the test.

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

- | | | | | | |
|-----|-----|-----|-----|-----|-----|
| 1. | (a) | (b) | (●) | (d) | (e) |
| 2. | (a) | (b) | (c) | (●) | (e) |
| 3. | (a) | (b) | (c) | (d) | (●) |
| 4. | (●) | (b) | (c) | (d) | (e) |
| 5. | (a) | (●) | (c) | (d) | (e) |
| 6. | (●) | (b) | (c) | (d) | (e) |
| 7. | (●) | (b) | (c) | (d) | (e) |
| 8. | (a) | (b) | (c) | (d) | (●) |
| 9. | (a) | (b) | (c) | (●) | (e) |
| 10. | (a) | (b) | (c) | (●) | (e) |

DO NOT WRITE IN THIS BOX!

Total multiple choice: _____

11. _____

12. _____

13. _____

14. _____

15. _____

Total: _____