11.

$$y' = 1 - 2\sin x.$$

This function is continuous everywhere and in particular on $[0, 2\pi]$, so the critical points are where $1 - 2\sin x = 0$; $\sin x = \dfrac{1}{2}$. The solutions are $x = \dfrac{\pi}{6} + 2k\pi$ and $x = \dfrac{5\pi}{6} + 2k\pi$ for any integer k. The solutions in the interval $[0, 2\pi]$ are $\dfrac{\pi}{6}$ and $\dfrac{5\pi}{6}$. Compute $y'(0) = 1 - 2\sin(0) = 1 > 0$ and $y'(2\pi) = 1 - 2\sin(2\pi) = 1 > 0$. Between $\dfrac{\pi}{6}$ and $\dfrac{5\pi}{6}$ lies for example $\dfrac{\pi}{2}$ and $y'\left(\dfrac{\pi}{2}\right) = 1 - 2\sin\left(\dfrac{\pi}{2}\right) = 1 - 2\cdot 1 = -1 < 0$. Hence y is decreasing on the interval $\left[\dfrac{\pi}{6}, \dfrac{5\pi}{6}\right]$.

The local maximum occurs at $\dfrac{\pi}{6}$. The point is $\left(\dfrac{\pi}{6}, \dfrac{\pi}{6} + 2\cos\left(\dfrac{\pi}{6}\right)\right) = \left(\dfrac{\pi}{6}, \dfrac{\pi}{6} + 2\dfrac{\sqrt{3}}{2}\right) = \left(\dfrac{\pi}{6}, \dfrac{\pi}{6} + \sqrt{3}\right)$. The book defines a local extremum to occur in the interior of any interval of definition, so the left hand end point is not a local maxima, even though the function is a maximum there. This point was not pursued in the grading unless you got the coordinates wrong. The left hand end point is $\big(2\pi, 2\pi + 2\cos(2\pi)\big) = \big(2\pi, 2\pi + 2\big)$

The local minimum occur at $\dfrac{5\pi}{6}$ and the point is $\left(\dfrac{5\pi}{6}, \dfrac{5\pi}{6} + 2\cos\left(\dfrac{5\pi}{6}\right)\right) = \left(\dfrac{5\pi}{6}, \dfrac{5\pi}{6} - 2\dfrac{\sqrt{3}}{2}\right) = \left(\dfrac{5\pi}{6}, \dfrac{5\pi}{6} - \sqrt{3}\right)$.

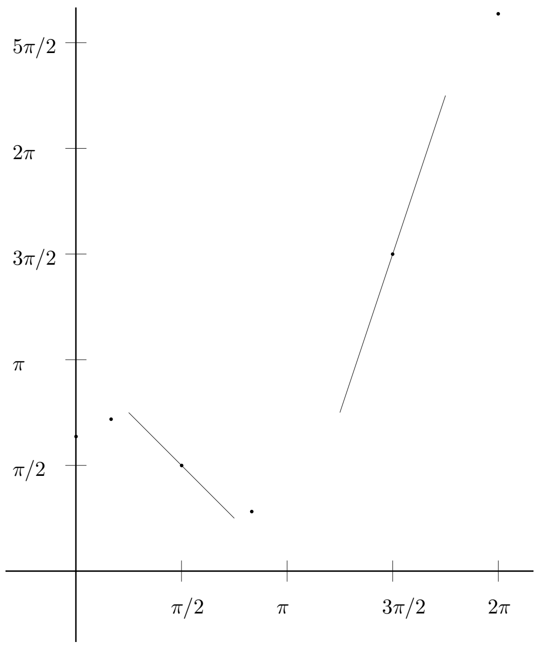
Again the right hand end point is not a local minimum by definition: its coordinates are $\big(0, 0 + 2\cos(0)\big) = (0, 2)$.

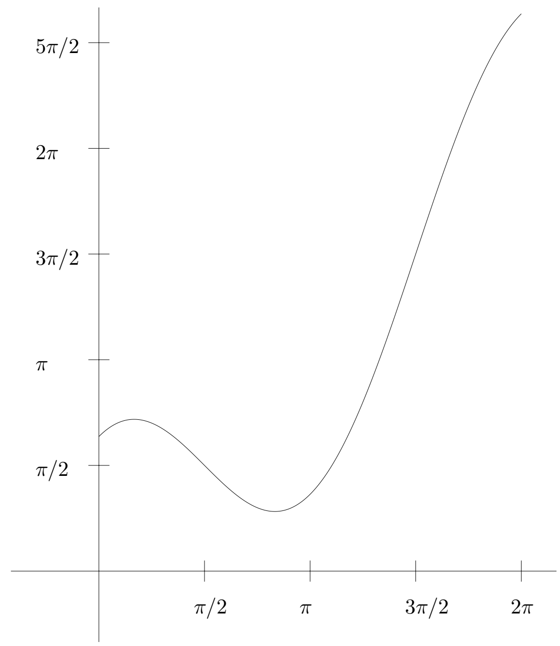
$$y'' = -2\cos x.$$

The second derivative is also continuous everywhere so we need to locate the zeros of $-2\cos x$ in the interval $[0, 2\pi]$. They are $\dfrac{\pi}{2}$ and $\dfrac{3\pi}{2}$. Compute $y''(0) = -2\cos(0) = -2 < 0$ and $y''(2\pi) = -2\cos(2\pi) = -2 < 0$. Between $\dfrac{\pi}{2}$ and $\dfrac{3\pi}{2}$ lies π and $y''(\pi) = -2\cos(\pi) = (-2)\cdot(-1) > 0$. Hence y'' is concave down on the intervals $\left[0, \dfrac{\pi}{2}\right]$ and $\left[\dfrac{3\pi}{2}, 2\pi\right]$.

Points of inflection occur at $\dfrac{\pi}{2}$ and $\dfrac{3\pi}{2}$. The corresponding points are $\left(\dfrac{\pi}{2}, \dfrac{\pi}{2} + 2\cos\left(\dfrac{\pi}{2}\right)\right) = \left(\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$ and $\left(\dfrac{3\pi}{2}, \dfrac{3\pi}{2} + 2\cos\left(\dfrac{3\pi}{2}\right)\right) = \left(\dfrac{3\pi}{2}, \dfrac{3\pi}{2}\right)$. The slopes are $y'\left(\dfrac{\pi}{2}\right) = 1 - 2\sin\left(\dfrac{\pi}{2}\right) = 1 - 2\cdot 1 = -1$ and $y'\left(\dfrac{3\pi}{2}\right) = 1 - 2\sin\left(\dfrac{3\pi}{2}\right) = 1 - 2\cdot(-1) = 3$.

The graph looks like





The left hand picture is just the axes with the points from our analysis plotted, together with the slopes from 2c). The right hand picture is the actual graph.

12.

(a) $s(t) = \int 96 - 32t dt = 96t - 16t^2 + C$. Using $s(0) = 256$, we obtain $C = 256$, so $s(t) = 96t - 16t^2 + 256$.

(b) Solve $s(t) = 0$. $0 = 96t - 16t^2 + 256 = -16(t^2 - 6t - 16) = -16(t-8)(t+2)$, so $t = 8$ or $t = -2$. $t = 8$ is the physically meaningful answer.

(c) The ball is at its highest point when velocity is 0 so we should solve $0 = v(t) = -32t + 96$. We obtain $0 = -32(t-3)$, so $t = 3$. When $t = 3$, $s(3) = 400$.

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13. The curves intersect when $x^2 + 2x + 3 = 2x + 4 \Rightarrow x^2 - 1 = 0 \Rightarrow x = \pm 1$. On the interval $[-1, 1]$, we have $2x + 4 \geq x^2 + 2x + 3$. Thus the area bounded by the two curves is given by

$$\int_{-1}^{1}[(2x+4)-(x^2+2x+3)]dx = \int_{-1}^{1}(-x^2+1)dx \;=\; -\frac{x^3}{3}+x\Big|_{-1}^{1}$$
$$= -\frac{1}{3}+1-(-\frac{-1}{3}-1) \;=\; 2-\frac{2}{3} \;=\; \frac{4}{3}\;.$$

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14. Denote the dimensions of the box by x,x and h. Then the cost function and volume are as follows: $c(x,y) = 6x^2 + 4(3xh) = 6x^2 + 12xh$ and $V = x^2h = 125$. We need to minimize the cost function using the volume as a constraint. Namely, substitute $h = \dfrac{125}{x^2}$ into the expression for the cost function resulting in:

$$C(x) = 6x^2 + 12x\frac{125}{x^2} = 6x^2 + \frac{1500}{x}.$$

Now, let's find x value where the minimum of the cost function occurs. We start by looking for critical points:

$$C'(x) = 12x - \frac{1500}{x^2} = 0.$$

This results in $x = 5$. Finally, we show that $x = 5$ is where a minima occurs by using second derivative test:

$$C''(x) = 12 + \frac{3000}{x^3}, C''(5) > 0.$$

Using the constraint yields $h = 5$.

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15. Let x be the base and y the height of the inscribed rectangle. We want to find x and y to maximize the area $A = xy$. The equation of the hypotenuse is $y = -2x + 20$, so

$$A(x) = x(-2x+20) = -2x^2 + 20x$$

$$A'(x) = -4x + 20$$

The only critical number is $x = 5$, and then $y = 10$. Since the derivative is positive for $x < 5$ and negative for $x > 5$, the value $A(5) = 50$ is the absolute maximum. (The equation $y = -2x + 20$ can also be found by using similar triangles, or by setting the sum of the areas of the two smaller triangles and the rectangle equal to the area of the large triangle.)
