1.(6 pts.) Determine the following limit

$$\lim_{x\to -1}\left(\frac{x^2+6x+5}{x^2+x}+\cos(x^2\pi)\right).$$

(a) -3 (b) 1 (c) -5 (d) -4 (e) 2

2.(6 pts.) Calculate the following limit

$$\lim_{x\to 3^-}\frac{x^2\mid x-3\mid}{x-3}.$$

(a) -9 (b) 9 (c) -3 (d) 3 (e) 6

3.(6 pts.) Find all points where the following function is discontinuous

$$f(x)=\begin{cases}\dfrac{(x-1)(x+2)}{(x^2-1)x} & x\neq 1\\ \dfrac{3}{2} & x=1\end{cases}.$$

(a) $x=0, x=-2, x=1$ (b) $x=-2, x=-1, x=1$

(c) $x=1, x=0$ (d) $x=-1, x=1, x=0$

(e) $x=0, x=-1$

4.(6 pts.) Let

$$f(x)=\sin(x)\cos(2x)+\sin(2x)\cos(x).$$

Find $f'(\pi)=$?

(a) -3 (b) -2 (c) 2 (d) 0 (e) -1

5.(6 pts.) Find the equation of the tangent line to the following curve at $(1,2)$:

$$f(x)=x^2-x+2.$$

(a) $y=-x-1$ (b) $y=-x+3$ (c) $y=x+1$ (d) $y=x-1$ (e) $y=2x$

6.(6 pts.) Let

$$f(x) = \frac{\sin x}{x} \ .$$

Find $f''(x) = ?$

(a) $\dfrac{-x^2 \sin x - 3x \cos x + 3 \sin x}{x^3}$ (b) $\dfrac{x^2 \sin x + 4x \cos x + 2 \sin x}{x^3}$

(c) $\dfrac{x^2 \sin x - 3x \cos x + 2 \sin x}{x^3}$ (d) $\dfrac{-x^2 \sin x - 2x \cos x + 2 \sin x}{x^3}$

(e) $\dfrac{-x^2 \sin x + 4x \cos x + 5 \sin x}{x^3}$

7.(6 pts.) If $y = (x+1)^3 + \sin(5x^3 + \sin x + \pi)$ what is $y'(0)$?

(a) $1/2$ (b) 4 (c) $5/2$ (d) $3/2$ (e) 2

8.(6 pts.) If $\sin(x+y) = y^2 \cos x$, what is y' at the point $(0,0)$?

(a) -1 (b) 2 (c) 0 (d) 4 (e) -2

9.(6 pts.) The mechanics at South Bend Automotive are reboring a 6-inch-deep cylinder to fit a new piston. The machine they are using increases the cylinder's radius one-thousandth of an inch every 3 minutes. How rapidly is the cylinder volume increasing when its radius is 3.5 in.?

(a) $\dfrac{24.5\pi}{1000}$ in.^{3}/min (b) $\dfrac{7\pi}{3000}$ in.^{3}/min (c) $\dfrac{14\pi}{1000}$ in.^{3}/min

(d) $\dfrac{73.5\pi}{1000}$ in.^{3}/min (e) $\dfrac{10.5\pi}{3000}$ in.^{3}/min

10.(6 pts.) Use differentials (or, equivalently, a linear approximation) to estimate $f(3.2)$ if $f(3) = 72$ and $f'(3) = 7$.

(a) 72.7 (b) 73.4 (c) 70.6 (d) 65 (e) 79

11.(6 pts.) Identify the x values of the local maxima and minima of the function $f(x) = (x+4)^5(x-3)^4$.

(a) local minimum at $x = 3$; local maxima at $x = -4$ and $x = -1/9$.

(b) local minimum at $x = -4$; local maximum at $x = 3$

(c) local minimum at $x = 3$; local maximum at $x = -4$

(d) local minimum at $x = 3$; local maximum at $x = -1/9$.

(e) local minima at $x = 3$ and $x = -4$; local maximum at $x = -1/9$.

12.(6 pts.) Consider the function $f(x) = \dfrac{x+1}{x^2+x+1}$. Which of the following statements is true? Note that $y = 0$ is a horizontal asymptote and draw a rough sketch of the graph after anaylyzing f'.

(a) f has no absolute maximum and an absolute minimum at $x = -2$.

(b) f has an absolute maximum at $x = 0$ and an absolute minimum at $x = -2$.

(c) f has an absolute maximum at $x = 0$ and no absolute minimum.

(d) f has an absolute maximum at $x = 0$ and an absolute minimum at $x = 2$.

(e) f has no absolute maximum or minimum.

13.(6 pts.) The value $x = c$ given by the Mean Value Theorem applied to the function $f(x) = x^3 + x - 1$ on the interval $[0, 2]$ is

(a) $x = 1$ (b) $x = \dfrac{\sqrt{3}}{3}$ (c) $x = \dfrac{2\sqrt{3}}{3}$ (d) $x = 2$ (e) $x = \sqrt{\dfrac{7}{6}}$

14.(6 pts.) If $y = 2x^2 - x^4$, which one of the following is true?

(a) There is just one inflection point.

(b) There are two local maxima and two inflection points.

(c) There are two local minima and two inflection points.

(d) There is one inflection point, one local minima and one local maxima.

(e) The line $y = 4x - 4$ is a slant asymptote.

15.(6 pts.) Find

$$\lim_{x\to-\infty}\frac{\sqrt{4x^4+7x+2}}{\sqrt[3]{27x^6+17}}$$

(a) $\dfrac{\sqrt{2}}{\sqrt[3]{17}}$ (b) ∞ (c) $-\frac{2}{3}$ (d) $-\infty$ (e) $\frac{2}{3}$

16.(6 pts.) A page of a book is to have a total area of 150 square inches, with 1 inch margins at the top and sides, and a 2 inch margin at the bottom. Find the dimensions in inches of the page which will have the largest print area.

(a) $3\sqrt{7}\times\dfrac{50}{\sqrt{7}}$ (b) $13\times 11\frac{7}{13}$ (c) 15×10 (d) 30×5 (e) $\dfrac{30}{\sqrt{3}}\times 5\sqrt{3}$

17.(6 pts.) Newton's method is to be used to find a root of the equation

$$x^3-x-1=0\ .$$

If $x_1=1$, find x_2.

(a) 1.35 (b) 1.50 (c) 1.75 (d) 3 (e) 0.95

18.(6 pts.) Express the limit below as a definite integral. Calculate that integral.

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{\pi}{4n}\sec^2\Big(\frac{i\pi}{4n}\Big)$$

(a) $-\pi$ (b) 1 (c) π (d) -1 (e) 0

19.(6 pts.) Find $f'(2)$ if $f(x)=\displaystyle\int_2^x\sqrt{1+t^3}\,dt$.

(a) 1 (b) 9 (c) 0 (d) 4 (e) 3

20.(6 pts.) A car racing on a straight road crosses the starting line with a velocity of 88 ft/sec. From this point on it accelerates at $\dfrac{60}{\sqrt{t+1}}$ ft/sec/sec. How fast in ft/sec will the car be going in 3 seconds?

(a) 208 (b) 328 (c) 244 (d) 292 (e) 152

21.(6 pts.) Find

$$\int_3^6 \frac{\pi}{x^2}\cos(\frac{\pi}{x})\ dx$$

(a) $\dfrac{\sqrt{3}-1}{2}$ (b) $\dfrac{\sqrt{3}+1}{2}$ (c) $\dfrac{1}{\sqrt{2}}$ (d) $\dfrac{1}{2}$ (e) 0

22.(6 pts.) A solid in space intersects the xy plane in the region between $x=0$ and $x=\pi$; bounded above by the graph of $y=\sin x$ and below by the graph of $y=-2\sin x$. Each cross section perpendicular to the x–axis is a rectangle whose short side is the line in the xy plane and whose long side is 2 times the length of the short side. Which integral below gives the volume?

(a) $18\displaystyle\int_0^\pi \sin x\ dx$ (b) $6\displaystyle\int_0^\pi \sin^2 x\ dx$ (c) $-\displaystyle\int_0^\pi \sin^2 x\ dx$

(d) $18\displaystyle\int_0^\pi \sin^2 x\ dx$ (e) $6\displaystyle\int_0^\pi \sin x\ dx$

23.(6 pts.) The plane region bounded below by the graph of $y=\sqrt{x}$ and above by the graph of $y=\sqrt[3]{x}$ is rotated about the line $x=5$. Which integral below gives the volume?

(a) $2\pi\displaystyle\int_0^1 \big(5-x\big)\cdot\big(\sqrt[3]{x}-\sqrt{x}\ \big)\ dx$ (b) $2\pi\displaystyle\int_0^1 \big(5-x\big)\cdot\big(\sqrt{x}-\sqrt[3]{x}\ \big)\ dx$

(c) $\pi\displaystyle\int_0^1 \big(5-\sqrt[3]{x}\ \big)^2-\big(5-\sqrt{x}\ \big)^2\ dx$ (d) $\pi\displaystyle\int_0^1 \big(5-\sqrt{x}\ \big)^2-\big(5-\sqrt[3]{x}\ \big)^2\ dx$

(e) $2\pi\displaystyle\int_0^1 \big(x-5\big)\cdot\big(\sqrt[3]{x}-\sqrt{x}\ \big)\ dx$

24.(6 pts.) Consider the plane region bounded by the graphs of $y=\sqrt{x}$, $y=0$, $x=0$ and $x=4$. Rotate this region about the line $y=-1$ and calcualte the volume.

(a) $\dfrac{61\pi}{3}$ (b) 28π (c) $\dfrac{28\pi}{3}$ (d) $\dfrac{56\pi}{3}$ (e) $\dfrac{112\pi}{3}$

25.(6 pts.) The function $f(x)=\sqrt[3]{x}$ is continuous on the interval $[1,8]$. Which number below is its average value on this interval?

(a) $\dfrac{5}{2}$ (b) $\dfrac{45}{28}$ (c) $\dfrac{5}{3}$ (d) $\dfrac{7}{5}$ (e) $\dfrac{45}{8}$

Name: _____

Instructor: ANSWER

Math 125, Final Exam$_1$, December 12, 2002

- Be sure that you have all 6 pages of the test.
- No calculators are to be used.
- The exam lasts for two hours.
- **When told to begin, remove this answer sheet and keep it under the rest of your test. When told to stop, hand in just this one page.**
- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.

Please mark your answers with an **X**! Do NOT circle them!

The dotted lines in the answer box indicate page breaks.

1.	(a)	(b)	(•)	(d)	(e)	15.	(a)	(b)	(c)	(d)	(•)
2.	(•)	(b)	(c)	(d)	(e)	16.	(a)	(b)	(•)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(•)	17.	(a)	(•)	(c)	(d)	(e)
4.	(•)	(b)	(c)	(d)	(e)	18.	(a)	(•)	(c)	(d)	(e)
5.	(a)	(b)	(•)	(d)	(e)	19.	(a)	(b)	(c)	(d)	(•)
6.	(a)	(b)	(c)	(•)	(e)	20.	(•)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(•)	21.	(•)	(b)	(c)	(d)	(e)
8.	(•)	(b)	(c)	(d)	(e)	22.	(a)	(b)	(c)	(•)	(e)
9.	(a)	(b)	(•)	(d)	(e)	23.	(•)	(b)	(c)	(d)	(e)
10.	(a)	(•)	(c)	(d)	(e)	24.	(a)	(b)	(c)	(•)	(e)
11.	(a)	(b)	(c)	(•)	(e)	25.	(a)	(•)	(c)	(d)	(e)
12.	(a)	(•)	(c)	(d)	(e)						
13.	(a)	(b)	(•)	(d)	(e)						
14.	(a)	(•)	(c)	(d)	(e)						

Final Exam: _____

Previous Total: _____

Course Total: _____