

Math 125 Final Exam
May 6, 2004

Name: _____

You need not find derivatives or integrals by the limit definition.

Please show your work.

You are taking this exam under the honor code.

1. (5 pts.) Let y be given by the formula $x^2 + y^3 = 2xy$. Find y' .

2. Let $f(x) = \frac{1}{2} \sin x + \cos x$.

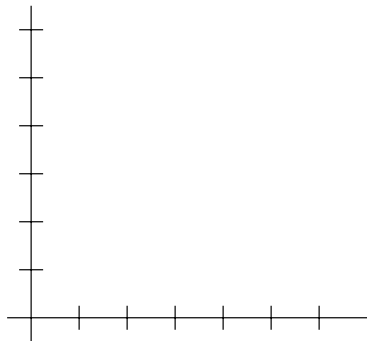
(a) (4 pts.) Find $f'(x)$.

(b) (5 pts.) What is $\int_0^\pi f(x)dx$?

3. (10 pts.) Let $f(x) = x^2 - 3x + 2$ and $g(x) = x - 1$. Find the area enclosed by f and g .

4. (9 pts.) Round cookies are baking in the oven. Assume each cookie is always an even 1 cm thick and always circular. As they bake, the volume of each cookie increases at a rate of 0.5 cm^3 per minute. How fast is the radius of a cookie increasing when that cookie has a radius of 2 cm?

5. (12 pts.) Use the shell method to find the volume of the solid obtained by rotating the following region about the y -axis.



6. The velocity of a particle at time t , in meters per second, is given by $v(t) = t^2 - 4$.

(a) (5 pts.) If the position of the particle at time $t = 1$ is $\frac{4}{3}$, find the position function for the particle.

(b) (4 pts.) What is the acceleration of the particle at time t ?

(c) (2 pts.) When does the particle achieve an acceleration of 6 m^2 per second?

7. For each of the following functions, find the limit of the function as x approaches infinity.

(a) (4 pts.) $h(x) = \frac{x^3+15x-100}{x^4+5}$

(b) (4 pts.) $g(x) = \frac{x^3}{2x^3+3x^2+x+12}$

(c) (4 pts.) $f(x) = \frac{2x^2+2x-5}{x}$

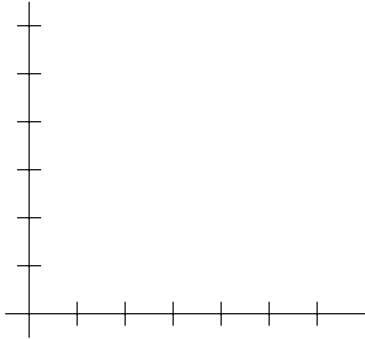
8. Given $f(x) = 1/x$, $g(x) = x^2 + 2x$, and $h(x) = x + 1$, find the following functions.

(a) (3 pts.) $g \circ f(x)$

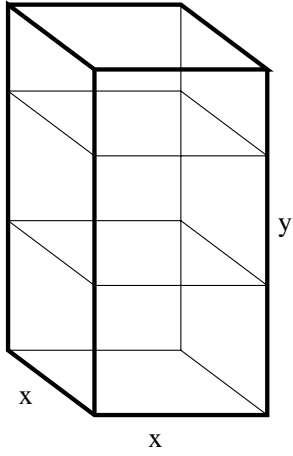
(b) (3 pts.) $f \circ h(x)$

9. (5 pts.) Find the average value of $f(x) = 2\sqrt{x}$ on the interval $[0, 4]$, and a value c in the interval such that $f(c)$ is the average value of f .

10. (12 pts.) Use the method of slicing to find the volume of the solid obtained by rotating the following region about the y -axis.



11. (10 pts.) A box with a square base is to be made to hold 10 ft^3 of material. The box will also contain two shelves parallel to the base (see figure). The material for the outside of the box costs \$4 per square foot, and the material for the shelves costs \$1 per square foot. If the base of the box is x by x meters, and the height is y , what should x and y be to minimize the cost of the box?



12. Let $f(x) = \frac{1}{2}x^4 - 3x^2 + 1$.

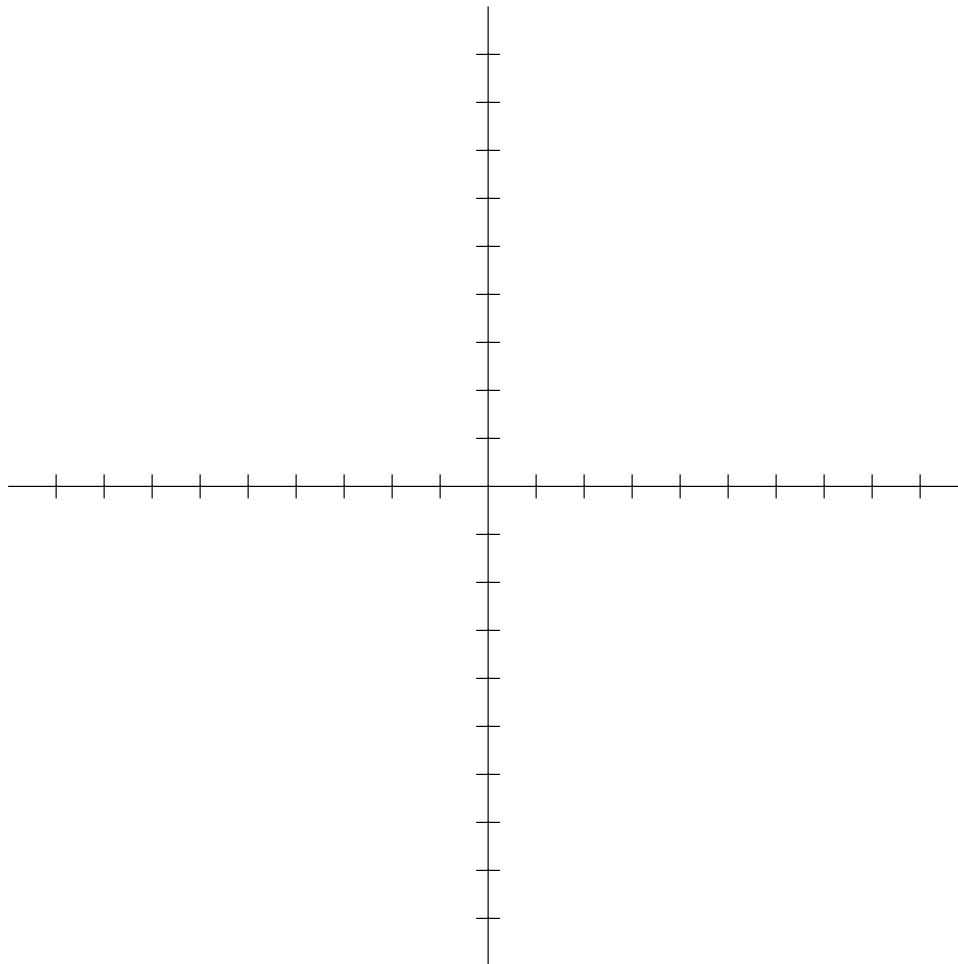
(a) (6 pts.) Find all critical points of $f(x)$.

(b) (4 pts.) On what intervals is f increasing or decreasing?

(c) (6 pts.) Find all potential inflection points of $f(x)$.

(d) (4 pts.) On what intervals is f concave up or concave down?

13. (6 pts.) Let $f(x) = \frac{3x^4 - 87x^2 + 300}{x^4}$. The following is a partial graph of f , showing all x -intercepts. Complete the graph by finding the behavior of f near $x = 0$ and as x approaches $\pm\infty$.



14. Evaluate the following limits or state why they do not exist.

$$f(x) = \begin{cases} \frac{x^2-1}{x+1} & \text{if } x \leq 2 \\ 2x & \text{if } x > 2 \end{cases}$$

(a) (5 pts.) $\lim_{x \rightarrow (-1)} f(x)$

(b) (5 pts.) $\lim_{x \rightarrow 2^-} f(x)$

(c) (5 pts.) $\lim_{x \rightarrow 2^+} f(x)$

(d) (2 pts.) $\lim_{x \rightarrow 2} f(x)$

15. (6 pts.) Evaluate the following integral.

$$\int_0^{\pi} 2x \cos(x^2) dx$$

16. (Extra Credit - 5 pts.) We used finite approximation and limits to define both derivatives and integrals. Choose one of those definitions and explain it in your own words (avoiding mathematical notation when possible).