Math 125 Test 1

February 6, 2004

Name:_____

You are taking this exam under the honor code.

1. Given the following function, find the limits below or explain why they do not exist.

$$f(x) = \begin{cases} 5x^2 + 3x + 12 & \text{if } x \leq 1 \\ 4 - x & \text{if } x > 1 \end{cases}$$

(a) (5 pts.) $\lim_{x \to 1^-} f(x)$

(b) (5 pts.) $\lim_{x \to 1^+} f(x)$

(c) (3 pts.) $\lim_{x \to 1} f(x)$

2. (8 pts.) Find $f'(5)$ for $f(x) = x^2 + 3x$.

3. Let $f(x) = \frac{x^2-1}{x-1}$.

   (a) Find the following values or say why they do not exist.

      i. (4 pts.) $f(1)$

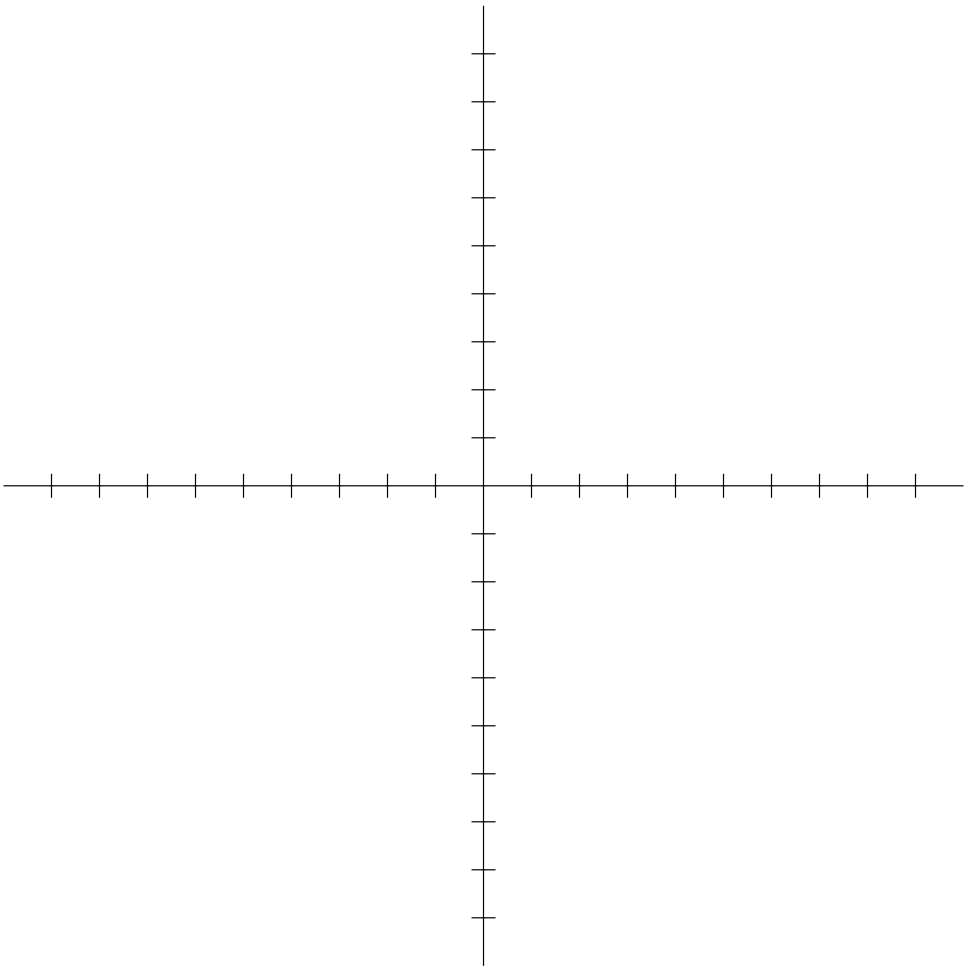
      ii. (6 pts.) $\lim_{x\to 1} f(x)$

   (b) (4 pts.) For what values of x is $f(x)$ continuous?

4. (4 pts.) Suppose $f(x)$ takes a measurement (in feet) as its input, and outputs a dollar amount. What are the units of $f'(x)$?

5. (8 pts.) Graph the following function:

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ 3x & \text{if } x > 1 \end{cases}$$



6. Let $f(x) = \frac{1}{x+1}$.

   (a) (2 pts.) What is the domain of $f(x)$?

   (b) (8 pts.) What is $f'(x)$?

   (c) (2 pts.) What is the domain of $f'(x)$?

7. (5 pts.) Suppose $f'(7) = 0.034$. What does the graph of $f(x)$ do at $x = 7$?

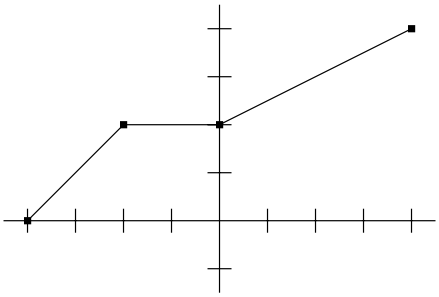
8. Given $f(x) = 1/x$ and $g(x) = x^2 + 2x$, find the following functions and state their domains.

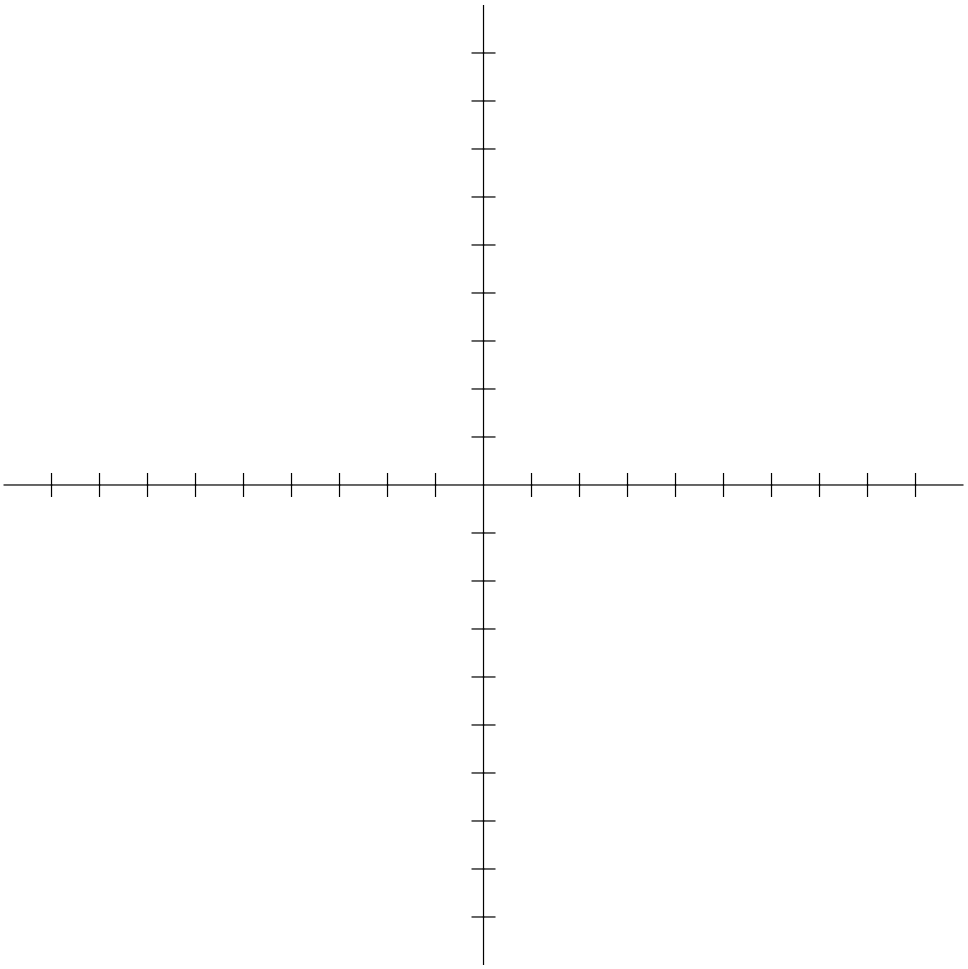
   (a) (5 pts.) $(f/g)(x)$

   (b) (7 pts.) $g \circ f(x)$

9. (8 pts.) Let $f(x) = x^4 - 10x^3 + 5$. Use the Intermediate Value Theorem (if applicable) to show there is a root of $f(x)$ in the interval $(0, 1)$.

10. (8 pts.) The graph of $f(x)$ is given. Draw the graph of $f(x/2)$.





11. (8 pts.) Suppose for $0 \leq x \leq 2$, the following inequality is true:

$$-2x + 6 \leq f(x) \leq x^2 - 4x + 7$$

Find $\lim_{x \to 1} f(x)$.

12. (Extra Credit - 3 pts.) Fill in the blanks: To prove that $\lim_{x \to a} f(x) = L$, we would need to show that for any $\epsilon > 0$, we can find $\delta > 0$ so that

$$0 < \_\_\_\_\_ < \delta$$

ensures

$$\_\_\_\_\_ < \epsilon.$$