Math 125 Midterm
March 5, 2004

Name: _____

You are taking this exam under the honor code.

You need not find derivatives by the definition. Please show your work.

1. (7 pts.) Let y be given by the formula $xy + y = x^2$. Find y'.

2. Let $f(x) = x^2(x+2)^5$.

   (a) (6 pts.) Find $f'(x)$.

   (b) (8 pts.) Find $f''(x)$.

3. Let $f(x) = 2x^3 + 3x^2 - 12x + 6$.

    (a) (7 pts.) Find all critical points of $f(x)$.

    (b) (7 pts.) Using your work from part (a), find the absolute maximum and minimum of f on the interval $[-2, 2]$.

4. (4 pts.) If $f(x) = 2|x| - x^2$, state where f is continuous and where it is differentiable.

5. (6 pts.) Suppose f is continuous on the interval $[0, 3]$, $f(0) = 1$, and for all x in $(0, 3)$, $0 \leq f'(x) \leq 1$. Give an inequality bounding the possible values for $f(3)$.

6. (8 pts.) Let $f(x) = (\sin \sqrt{x})^3$. Find $f'(x)$.

7. (5 pts.) Evaluate the following limit or state why it does not exist.

$$\lim_{x \to 5} \frac{x^2 - 3x - 10}{x - 5}$$

8. The position in meters of a particle at time t seconds is given by

$$f(t) = \frac{1}{3}t^3 - 2t^2 + 25.$$

(a) (5 pts.) What is the velocity of the particle at time t?

(b) (5 pts.) What is the acceleration of the particle at time t?

(c) (4 pts.) When does the particle achieve a velocity of -4 meters per second?

9. Let $y = f(x) = x^2 + x - 1$, and consider x as it changes from 1 to 2.

   (a) (6 pts.) Find the differential dy for this change in x.

   (b) (4 pts.) Find the actual change Δy for this change in x.

10. (10 pts.) A spherical lump of bread dough is increasing in volume at a rate of 1/2 cubic inch per minute. Assume the dough always maintains its spherical shape. What is the rate of change of the radius of the dough at the time when the diameter is 6 inches?

11. (8 pts.) Find $f'(x)$ for $f(x) = \sin(2x)\cos(x)$. What is $f'(\frac{\pi}{4})$?