Math 125 Test 3

April 7, 2004

Name: ____

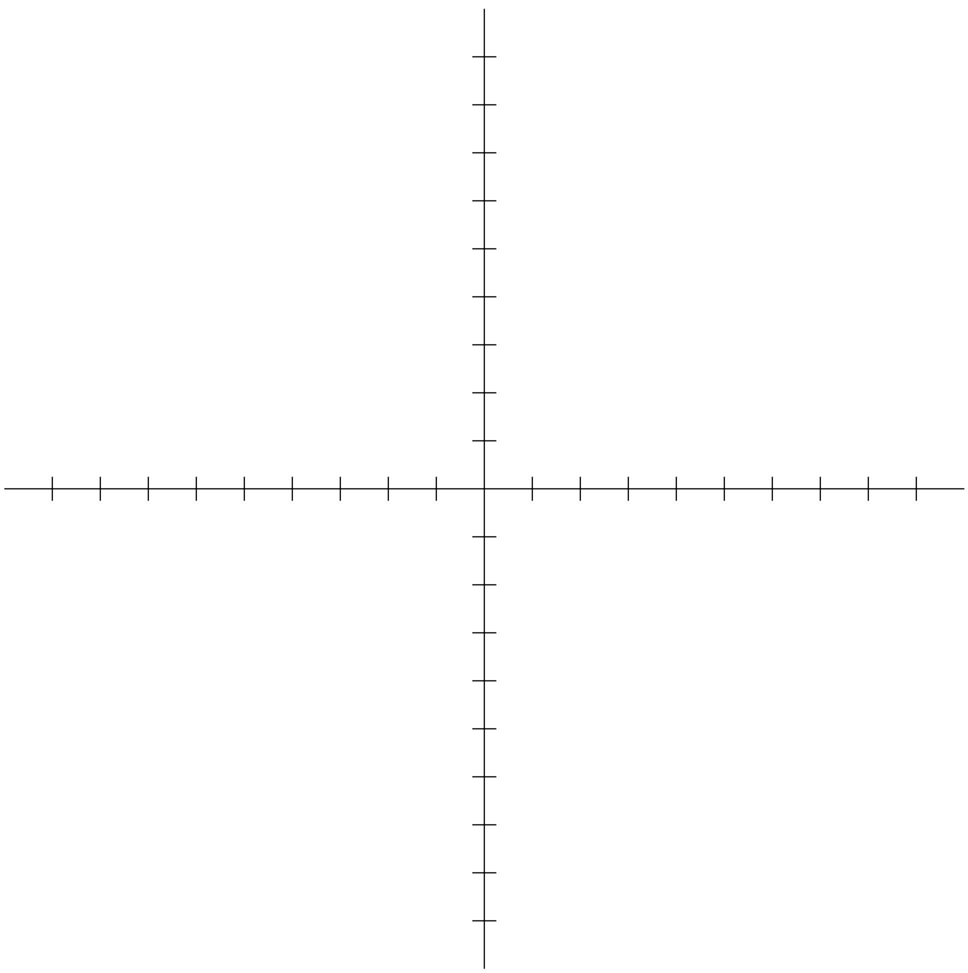
You are taking this exam under the honor code.

You need not find derivatives by the definition. Please show your work.

1. (7 pts.) The acceleration of a particle at time t is given by $a(t) = 2t + 2$. If the velocity of the particle at time 0 is -3, find the velocity function for the particle.

2. (12 pts.) Let $f(x) = \frac{(x+4)(x-5)}{x^3}$. Find the vertical and horizontal asymptotes of f (if they exist) and the intercepts. Using that and the following information, sketch a rough graph of f.

- $f'(x)$ is positive on the intervals $(-6.8, 0)$ and $(0, 8.8)$, and negative on $(-\infty, -6.8)$ and $(8.8, \infty)$.
- $f''(x)$ is positive on the intervals $(-9.6, 0)$ and $(12.6, \infty)$, and negative on $(-\infty, -9.6)$ and $(12.6, \infty)$.



3. (15 pts.) For each of the following functions, find the limit of the function as x approaches infinity. If the function has a slant asymptote, find the equation of the asymptote.

   (a) $g(x) = \frac{4x^3}{2x^3+3x^2+x-15}$

   (b) $f(x) = \frac{2x^2+5}{x-3}$

   (c) $h(x) = \frac{x^3-2x+10}{x+5}$

4. (12 pts.) A box with a square base is to be made to hold 16 m^3 of material. The box has to be made to stack, so the materials for the top and bottom cost \$10 per square meter, while the sides only cost \$5 per square meter. If the base of the box is x by x meters, and the height is y, what should x and y be to minimize the cost of the box?

5. Let $f(x) = 2x + \cos x$.

(a) (4 pts.) Find the most general antiderivative of f.

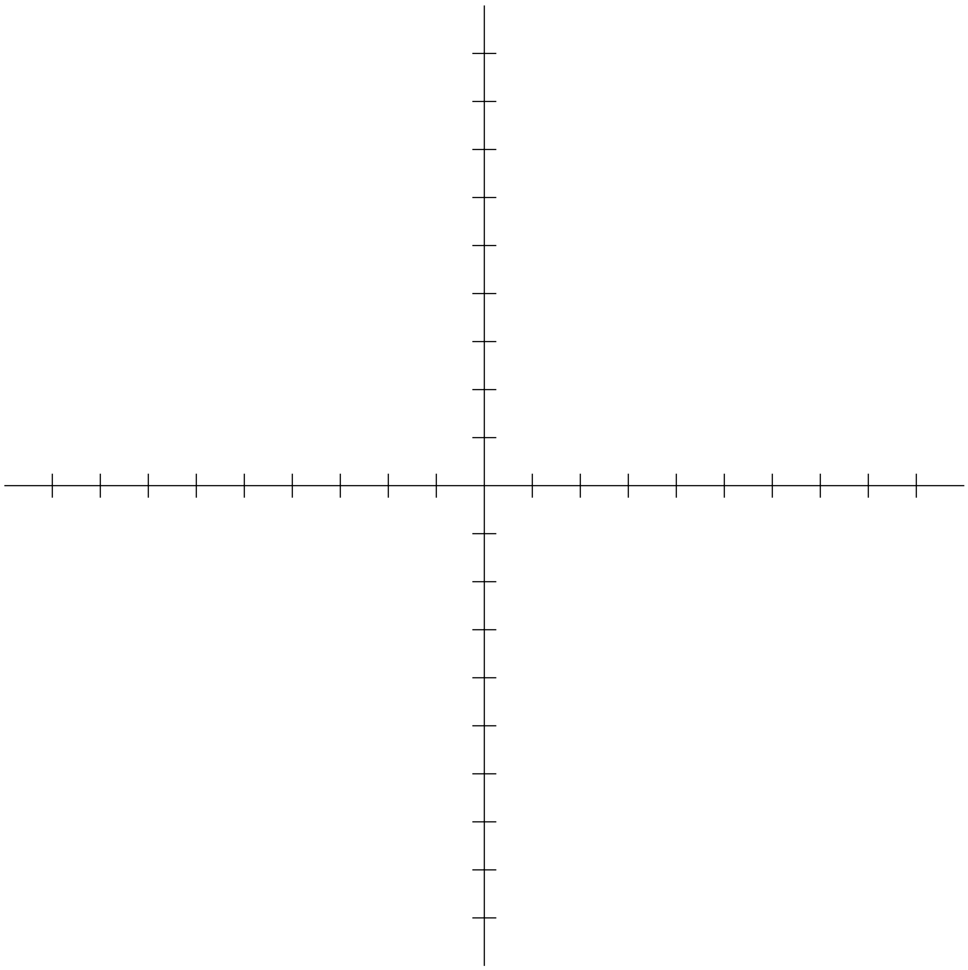
(b) (4 pts.) Use part (a) and the Fundamental Theorem of Calculus to evaluate $\int_0^{\pi} f(x)dx$.

6. (4 pts.) Find $\frac{d}{dx} \int_0^x (\sin(2t + 5) + t^4)dt$.

7. (9 pts.) Find the Riemann sum approximation for the area under $f(x) = x^2 - 1$ on the interval $[1, 5]$, using $\Delta x = 1$. You may use either the left or right endpoint method, but you must state which method you use.

8. (16 pts.) Let $f(x) = x^4 - 6x^2 + 5$. Find the extreme points of f, its intervals of increase and decrease, the inflection points of f, and its intervals of positive and negative concavity, and use them to sketch a graph of f. A Cartesian plane is provided on the next page. To assist you, below are some values of f, f', and f'', as well as the approximate numerical values of some square roots.

x	$f(x)$	x	$f'(x)$	x	$f''(x)$	x	$\sqrt{x}$
-4	165	-4	-208	-5	288	2	1.4
$-\sqrt{3}$	-4	-2	-8	-3	96	3	1.7
-1	0	-1	8	0	-12	5	2.2
0	5	1	-8	3	96	7	2.6
1	0	2	8	5	288		
$\sqrt{3}$	-4	4	208				
4	165						



9. (12 pts.) Using the Riemann sum definition of the definite integral, find $\int_0^1 \frac{x^2}{2}dx$. Some sum formulas are given.

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

10. (5 pts.) Suppose $\int_{-2}^{3} f(x)dx = 12$ and $\int_{3}^{-4} f(x)dx = -15$. What is $\int_{-4}^{-2} f(x)dx$?

11. (3 pts.) Extra credit: If $u = x^2 - 1$, find

$$\frac{d}{dx}\int_{0}^{u}(t^2 + t + 1)dt.$$