1. $y = \ln\left(\frac{x}{1+x}\right)$. Then $\frac{dy}{dx} = ?$

(A) $\frac{1}{x + x^2}$ (B) $\frac{1}{x+1}$ (C) $\frac{x+1}{x}$

(D) $\frac{x}{x^2 - 1}$ (E) $(x+1)^2$

2. Suppose $f(x) = x^x$. Then $f'(e) = ?$

(A) e^e (B) $e + \ln(e)$ (C) $e^{\ln(e)}$ (D) $2e^e$ (E) e^{1-e}

3. Suppose g is a function satisfying $g(5) = 0$, $\frac{dg}{dx}(5) = 3$.
Suppose g^{-1} is the inverse function of g. Then $\left(\frac{d}{dx}\, g^{-1}\right)(0) = ?$

(A) 5 (B) 3 (C) 0 (D) $\frac{1}{5}$ (E) $\frac{1}{3}$

4. Find the area of the shaded region at right

$$y = \frac{\ln(x)}{x}$$

(A) $e^2 - 1$ (B) $(e - 1)\ln(2)$

(C) 2 (D) $\ln(2)$

(E) $\ln(e^2 - 1)$

5. You invest $y(0)$ dollars in a bank at time $t = 0$. It grows rapidly, and after t years its value is $y(t)$ dollars. You know that

$$\frac{dy}{dt} = (.2)y$$

In how many years will your money double?

(A) $2e^{-2}$ (B) $\ln(.2)$ (C) $5\ln(2)$

(D) $2e$ (E) $\ln(2)$

6. $\lim_{x \to 0^+} \left(\frac{3x + 1}{x} - \frac{1}{\sin x} \right) = ?$

(A) ∞ (B) 3 (C) 2 (D) 1 (E) 0

7. Which of the following functions grows fastest as x goes to $+\infty$?

(A) x^2 (B) e^x (C) 2^x (D) x^e (E) $\ln(x)$

8. $\tan(\sin^{-1}(x)) = ?$

(A) $\frac{\sin^{-1}(x)}{\cos^{-1}(x)}$ (B) $\frac{1}{1+x^2}$ (C) $\frac{1}{\sqrt{1-x^2}}$

(D) $\sec(x)$ (E) $\frac{x}{\sqrt{1-x^2}}$

9. $\int_0^2 \frac{dx}{4+x^2} = ?$

(A) π (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) ∞ (E) $\frac{\pi}{8}$

10. Let $y = \ln(\sinh x)$. Compute $\frac{dy}{dx}$ when $x = 1$.

(A) 0 (B) 1 (C) $2e$ (D) $\frac{e + e^{-1}}{e - e^{-1}}$ (E) $\frac{e - e^{-1}}{2e}$

11. Suppose $y(x)$ satisfies: $y' + P(x)y = Q(x)$. Let $\nu(x) = e^{\int P(x)dx}$

Then $y = ?$

(A) $\nu(x) \int P(x)\, \nu(x)\, dx$ (B) $\frac{1}{\nu(x)} \int P(x)\, \nu(x) dx$

(C) $\frac{1}{\nu(x)} \int Q(x)\, \nu(x) dx$ (D) $\nu(x) \int \{Q(x)/\nu(x)\}\, dx$

(E) $\nu(x) \int Q(x)\, \nu(x)\, dx$

12. $\int_0^{\pi/6} 2\sqrt{\frac{1 + \cos(4x)}{2}}\, dx = ?$

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{\pi}{6}$ (C) 1 (D) $\sqrt{2}$ (E) $\sqrt{3}$

13. Suppose $\frac{dy}{dx}$ is the derivative of the function $y = \sin^{-1}(x)$.

Prove that $\frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}}$.