1. $\displaystyle\int_{1}^{e} 6x^5 \ln(x^2)\ dx = ?$

(A) $5e^6 - 1$

(B) $\frac{1}{3}\left[\ 5e^6 + 1\ \right]$

(C) $5e^6 + 1$

(D) $\frac{1}{3}\left[\ 5e^6 - 2\ \right]$

(E) $2e^6$

2. Which of the following fractions occurs as a summand in the partial fraction decomposition of $\dfrac{10x - 4}{x^3 - 4x}$?

(A) $\dfrac{-4}{x - 2}$ (B) $\dfrac{-3}{x - 2}$ (C) $\dfrac{2}{x - 2}$ (D) $\dfrac{3}{x - 2}$ (E) $\dfrac{-1}{x - 2}$

3. $\displaystyle\int_{2}^{3} \frac{x^3 + x^2 + 1}{x^2 - x}\ dx\ =\ ?$

(A) $\frac{1}{2} + \ln 3$

(B) $4 + \ln \frac{27}{2}$

(C) $8 + 3\ln \frac{3}{2}$

(D) $\frac{1}{2} + \ln \frac{27}{2}$

(E) $8 + \ln \frac{27}{2}$

4. Suppose the appropriate trigonometric substitution is made in the following integral. What trigonometric integral is obtained from making this substitution?

$$\int \frac{(1 - x^2)^{5/2}}{x^6}\ dx$$

(A) $\int \cot^6 \theta\ d\theta$

(B) $\int \cot^6 \theta \cos^5 \theta\ d\theta$

(C) $\int \tan^5 \theta \csc \theta\ d\theta$

(D) $\int \cot^5 \theta \csc \theta\ d\theta$

(E) $\int \tan^6 \theta\ d\theta$

5. $\displaystyle\int_{-1}^{0} \frac{dx}{x^2 + 2x + 2} = ?$

(A) 0

(B) $\dfrac{\pi}{6}$

(C) $\dfrac{\pi}{4}$

(D) $\dfrac{\pi}{3}$

(E) $\dfrac{\pi}{2}$

6. $\displaystyle\int_{1}^{3} \frac{2x}{(x^2 - 1)^{1/2}}\, dx = ?$

(A) Diverges

(B) $\sqrt{2}$

(C) $2\sqrt{2}$

(D) $3\sqrt{2}$

(E) $4\sqrt{2}$

7. Which of the following improper integrals converge and which diverge?

1. $\displaystyle\int_{2}^{\infty} \frac{1}{e^x}\, dx$
2. $\displaystyle\int_{2}^{\infty} \frac{1}{x + 4}\, dx$
3. $\displaystyle\int_{2}^{\infty} \frac{1}{\sqrt{x + 1}}\, dx$

(A) (1) converges, (2) and (3) diverge
(B) (1) and (3) converge, (2) diverges
(C) none converge, (1) and (2) and (3) diverge
(D) (2) and (3) converge, (1) diverges
(E) (3) converges, (1) and (2) diverge

8. $\displaystyle\lim_{n \to \infty} \frac{2n + (-1)^n + \cos n}{n} = ?$

(A) Diverges   (B) 0   (C) 2   (D) 3   (E) 4

9. $\lim\limits_{n\to\infty} \dfrac{\ln\,(n+1)}{\sqrt{n}} = ?$

(A) Diverges   (B) 1   (C) $\frac{1}{2}$ (D) 0   (E) $\ln 2$

10. $\sum\limits_{n=0}^{\infty} \dfrac{(-1)^n}{2 \cdot 3^n} = ?$

(A) 3   (B) $\frac{3}{8}$ (C) $-\frac{1}{3}$ (D) $\frac{3}{2}$ (E) $-\frac{1}{6}$

11. Which of the following infinite series converge and which diverge?

(1) $\sum\limits_{n=1}^{\infty} \dfrac{1}{(2n)!}$ (2) $\sum\limits_{n=1}^{\infty} \dfrac{1}{n\sqrt{n}}$ (3) $\sum\limits_{n=1}^{\infty} \dfrac{1}{n}$ (4) $\sum\limits_{n=1}^{\infty} \left(\dfrac{3}{2}\right)^n$

(A) (1) and (4) converge, (2) and (3) diverge
(B) (1) and (2) converge, (3) and (4) diverge
(C) (1) converges, (2) and (3) and (4) diverge
(D) (2) converges, (1) and (3) and (4) diverge
(E) (2) and (3) converge, (1) and (4) diverge

12. Which of the following infinite series converge and which diverge?

$$\sum_{n=1}^{\infty} \ln\left(\frac{1}{n^2}\right) \qquad (2) \ \sum_{n=1}^{\infty} \frac{2 + \cos n}{n^4} \qquad (3) \ \sum_{n=1}^{\infty} \frac{n^2 + 2}{n^4 - n^{1/3}}$$

(A) (1) and (2) converge, (3) diverges

(B) (2) and (3) converge, (1) diverges

(C) (1) converges, (2) and (3) diverge

(D) (3) converges, (1) and (2) diverge

(E) (1) and (2) and (3) converge, none diverge

13. Suppose the Integral Test is applied to the following infinite series. What is the best information that this test provides?

$$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$$

(A) diverges

(B) converges and the test gives an upper bound 1

(C) converges and the test gives an upper bound 2

(D) converges and the test gives an upper bound 3

(E) Converges and the test gives an upper bound 4

14. Suppose the Ratio Test is applied to each of the following infinite series. What information does this test provide?

(1) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$ (2) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 + n}{2^n}$ (3) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{2^n}$

(A) (2) converges, (3) diverges, no information about (1)

(B) (2) & (3) converge, (1) diverges

(C) (1) & (2) diverge, no information about (3)

(D) (1) converges, (2) & (3) diverge

(E) (2) & (3) converge, no information about (1)

15. Prove that $\displaystyle\lim_{n \to 0} \left(1 + \frac{c}{n}\right)^n = e^c$ for any real number c.

Please record a copy of your proof on the front page.