

1. What is the coefficient of  $x^2$  in the Maclaurin series generated by the function  $\ln(1 + \sin x)$ ?

- (A) 1      (B)  $\frac{1}{2}$       (C) 2      (D)  $-\frac{1}{2}$       (E) - 2

2. What is the sum of the series

$$1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \dots + (-1)^n \frac{(\ln 2)^n}{n!} + \dots ?$$

- (A)  $2e$       (B)  $e^{-2}$       (C)  $\frac{1}{2}$       (D)  $\frac{\pi}{2}$       (E) 2

3. What is the radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{3^n x^n}{n!} ?$$

- (A)  $\frac{1}{3}$       (B)  $e$       (C) 1      (D) 0      (E)  $\infty$

4. When  $|x| < 0.3$ , which of the numbers below is the best estimate of the error in the approximation  $\sin x \approx x$  ?

- (A) 0.0045      (B) 0.0090      (C) 0.0270  
(D) 0.0009      (E) 0.0027

5. The Taylor's Formula gives the following identity

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!} x^2 + \dots + \frac{f^{(n)}(0)}{n!} x^n + R_n(x)$$

What is the expression for  $R_n(x)$ ?

- (A)  $R_n(x) = \frac{f^{(n+1)}(0)}{(n+1)!} x^{n+1}$   
(B)  $R_n(x) = \frac{f^{(n)}(c)}{n!} x^n$  for some  $c$  between 0 and  $x$ .  
(C)  $R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!} x^{n+1}$  for some  $c$  between 0 and  $x$   
(D)  $R_n(x) = f^{(n+1)}(c) x^{n+1}$  for some  $c$  between 0 and  $x$   
(E) None of the above

6. Which of the following is the Maclaurin series for

$$\sinh x = \frac{e^x - e^{-x}}{2} \quad ?$$

(A)  $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots + \frac{x^n}{n!} + \dots$

(B)  $1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \dots$

(C)  $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$

(D)  $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$

(E)  $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$

7. Express the integral  $\int_0^1 \frac{\sin x}{x} dx$  by an infinite series.

8. Does the series  $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n}$  converge? If so, is the convergence absolute or conditional? Explain your answer.

9. For which  $x$  does the series  $\sum_{n=2}^{\infty} \frac{x^n}{\ln n}$  converge ?

For which  $x$  is the convergence absolute? For which  $x$  is the convergence conditional? Explain your answer.

10. Find the Taylor polynomial of order 4 generated by  $f(x) = e^x \sin x$  at 0.