1. The function $f(x) = \ln x + \arctan x$ has an inverse function $g(x)$.

Moreover: $f(1) = \frac{\pi}{4}$. What is $\frac{dg}{dx}\left(\frac{\pi}{4}\right)$?

(A) $\frac{2}{3}$ (B) $\frac{2}{\sqrt{\pi}}$ (C) $\frac{1}{2}$ (D) $\frac{2}{e}$ (E) 1

2. Suppose $y = \frac{e^{2x}}{1 + e^{2x}}$. What is $\frac{dy}{dx}$ when $x = 0$?

(A) $2e$ (B) e^{-2} (C) $\frac{1}{2}$ (D) $\frac{\pi}{2}$ (E) 2

3. What is $\int_1^2 \frac{dx}{\sqrt{-x^2 + 2x}}$?

(A) $-\frac{\pi}{2\sqrt{2}}$ (B) $\frac{\pi}{2}$ (C) $-\frac{\pi}{2}$ (D) $\frac{\pi}{\sqrt{2}}$ (E) $\frac{\pi}{2\sqrt{2}}$

4. In the partial fraction decomposition of the function, what is the numerator of the term whose denominator is x^2 ?

$$\frac{x^2 + 2}{x^2(x + 1)}$$

(A) -2 (B) 2 (C) -1 (D) 1 (E) 0

5. On the interval $(0, \infty)$, what is solution of the following initial value problem ?

$$\frac{dy}{dx} = \frac{3x^3 + 1}{xe^y}$$

$$y(1) = \ln 2.$$

(A) $e^y = x^3 + \ln(x) + 1$

(B) $e^y = x^3 + \ln(2x)$

(C) $y = \ln(x^3) + \ln(\ln(x)) + 2$

(D) $y = \ln(x^3) + \ln(\ln(x)) + \ln 2$

(E) $e^y = x^3 + \ln(x)$

6. Compute $\int_0^{\pi} x \cos x \, dx$.

(A) 1 (B) -1 (C) 2 (D) -2 (E) 0

7. Evaluate the following integral:

$$\int_{0}^{\pi/2} \cot \theta \, d\theta$$

(A) 0 (B) – 1 (C) $\ln \frac{\pi}{2}$ (D) 1 (E) diverges

8. How many of the following series are convergent?

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} \qquad \sum_{n=0}^{\infty} \frac{3^n}{n!} \qquad \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \qquad \sum_{n=0}^{\infty} \frac{3^n}{7^n}$$

(A) one (B) two (C) three (D) four (E) none

9. Which of the following statements is true of the series $\sum_{n=2}^{\infty} \frac{1}{\ln(n^n)}$?

(A) The series diverges by the integral test
(B) The series converges by the integral test
(C) The series diverges by the ratio test
(D) The series converges by the ratio test
(E) None of above

10. What is the radius of convergence of the following power series?

$$\sum_{n=1}^{\infty} \frac{x^n}{(2n)!}$$

(A) ∞ (B) 0 (C) 1 (D) ln 2 (E) 2

11. The first several terms of the Maclaurin series generated by the function f(x) are:

$$f(x) = 1 + x - \frac{1}{2}x^2 + \frac{1}{7}x^3 - \frac{1}{11}x^4 + \dots$$

What is $f^{(3)}(0)$?

(A) $\frac{1}{21}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{7}$ (D) $\frac{6}{7}$ (E) $-\frac{12}{11}$

12. What is the coefficient of x^2 in $(1 + x)^{100}$?

(A) 19800 (B) 4950 (C) $\frac{199}{2}$ (D) 9900 (E) 199

13. What is the slope of the line tangent to the curve $x = 1 - \cos t$, $y = \sin t - t$ at the point corresponding to $t = \frac{\pi}{3}$?

(A) 1 (B) $\frac{\pi}{2}$ (C) $-\frac{1}{2}$ (D) $\frac{\sqrt{3}}{2}$ (E) $-\frac{1}{\sqrt{3}}$

14. The following is the equation, in polar coordinates, of a circle:

$$r = \sin\theta \text{ .}$$

Find the rectangular coordinates of the center of this circle.

(A) $\left(\frac{\pi}{2}, 0\right)$ (B) (1, 0) (C) $\left(0, \frac{\pi}{2}\right)$ (D) $\left(0, \frac{1}{2}\right)$ (E) (0, – 1)

Partial Credit Problem

15. What is $\lim_{x \to \infty} \left(\frac{x + 1}{x - 1} \right)^x$?

16. What is $\int_1^\infty \frac{1}{x^2 + x} \, dx$?

17. Express the following integral as a power series,

$$\int_0^{0.1} x^2 e^{-x^2} \, dx$$

18. What are the four points of intersection of the following pair of curves ?

$r = \cos^2 \theta$

$r = \frac{1}{4}$

19. Prove the following theorem.

<u>Theorem</u>: $\lim_{x \to \infty} \left(1 + \frac{c}{n} \right)^n = e^c$

20. Prove the following theorem.

<u>Theorem:</u> Suppose f is a nonnegative continuous decreasing function on the interval $(1, \infty)$ and $\int_1^\infty f(x) \, dx < \infty$. Suppose $a_k = f(k)$ for $k = 1, 2, 3, \ldots$. Then $\sum_{k=1}^\infty a_k$ converges and $\sum_{k=1}^\infty a_k \leq a_1 + \int_1^\infty f(x) \, dx$