§6.1

#13. $y = x^2 + 1$, $x^2 = y - 1$, $x = \pm\sqrt{y-1}$, so $f^{-1}(x) = \sqrt{x-1}$ and we want the plus sign because the domain of f is $x \geq 0$.

#15. $y = x^3 - 1$, $x^3 = y + 1$, $x = \sqrt[3]{y+1}$, so $f^{-1}(t) = \sqrt[3]{t+1}$.

#23. $y = \frac{1}{x^2}$, $x^2 = \frac{1}{y}$, $x = \frac{\pm 1}{\sqrt{y}}$, so $f^{-1}(x) = \frac{1}{\sqrt{x}} = x^{-1/2}$ since the domain of f is $x > 0$. The domain of f^{-1} is all $x > 0$; the range of f^{-1} is all $x > 0$. These follow from the corresponding results for f. For f, we were told that the domain is $x > 0$. This gives the range of f^{-1}. By definition the range consists of all numbers y so that we can solve the equation $\frac{1}{x^2} = y$ with $x > 0$. This in turn is all y, $y > 0$, which gives the range of f and the domain of f^{-1}.

#31. $f(x) = x^3 - 3x^2 - 1$, $x \geq 2$; find $\frac{df^{-1}}{dx}$ at $x = -1$. Formula: $\frac{df^{-1}}{dx}(a) = \frac{1}{f'\left((f^{-1}(a)\right)}$. We want $\frac{df^{-1}}{dx}(-1)$ so what is $f^{-1}(-1)$? Equivalently, solve $f(a) = -1$ and we are told that the answer is 3. Since $f'(x) = 3x^2 - 6x$, $\frac{df^{-1}}{dx}(-1) = \frac{1}{3(3^2) - 6 \cdot 3} = \frac{1}{9}$.

#33. This problem is another application of the formula in the last problem. Since $(2, 4)$ is on the graph of f, $(4, 2)$ is on the graph of f^{-1}, or $f^{-1}(4) = 2$. The slope of f at 2 is $\frac{1}{3}$ and you may proceed either of two ways. Algebraic: $f'(2) = \frac{1}{3}$; $\frac{df^{-1}}{dx}(4) = \frac{1}{f'(2)}$ since $f^{-1}(4) = 2$. Hence $\frac{df^{-1}}{dx}(4) = 3$. Geometric: 1 equals the slope of a line times the slope of that line reflected in $y = x$. Hence $\left(\frac{df^{-1}}{dx}(4)\right) \cdot f'(2) = 1$.

#43. $f(x) = (1-x)^3$; $f'(x) = -3(1-x)^2$. Hence $f'(x) \leq 0$ and is equal to 0 only if $x = 1$. Hence f is decreasing and therefore one to one. $\frac{df^{-1}}{dx}(x) = \frac{1}{f'\left(f^{-1}(x)\right)} = \frac{1}{-3\left(1 - f^{-1}(x)\right)^2}$. To continue, we need a formula for f^{-1}: $y = (1-x)^3$, $\sqrt[3]{y} = 1 - x$, $x = 1 - \sqrt[3]{y}$, so $f^{-1}(x) = 1 - \sqrt[3]{x}$. Hence $\frac{df^{-1}}{dx}(x) = \frac{1}{-3\left(1 - (1 - \sqrt[3]{x}\,)\right)^2} = \frac{1}{-3(\sqrt[3]{x}\,)^2} = \frac{1}{-3x^{2/3}} = -\frac{1}{3}x^{-2/3}$.