§6.2

#1. a) $\ln 0.75 = \ln\frac{3}{4} = \ln 3 - \ln 4 = \ln 3 - \ln 2^2 = \ln 3 - 2\ln 2$.
b) $\ln(\frac{4}{9}) = \ln\big((\frac{2}{3})^2\big) = 2(\ln\frac{2}{3}) = 2(\ln 2 - \ln 3)$.
c) $\ln\frac{1}{2} = -\ln 2$.
d) $\ln\sqrt[3]{9} = \ln 9^{1/3} = \frac{1}{3}\cdot\ln 9 = \frac{1}{3}\ln(3^2) = \frac{2}{3}\ln 3$.
e) $\ln 3\sqrt{2} = \ln(3^{1/2}) = \frac{1}{2}\cdot\ln 3$.
f) $\ln\sqrt{13.5} = \ln\sqrt{\frac{27}{2}} = \ln\big((\frac{27}{2})^{1/2}\big) = \frac{1}{2}\cdot\ln(\frac{27}{2}) = \frac{1}{2}\big(\ln 27 - \ln 2\big) = \frac{1}{2}\big(\ln 3^3 - \ln 2\big) = \frac{1}{2}\big(3\ln 3 - \ln 2\big)$.

#5. $\frac{dy}{dx} = \frac{\frac{d(3x)}{dx}}{3x} = \frac{3}{3x} = \frac{1}{x}$.

#11. $\frac{dy}{d\theta} = \frac{\frac{d(\theta+1)}{dx}}{\theta+1} = \frac{1}{\theta+1}$.

#15. $\frac{dy}{dt} = \frac{dt}{dt}\cdot(\ln t)^2 + t\cdot\frac{d(\ln t)^2}{dt} = (\ln t)^2 + t\cdot\big(2(\ln t)^{2-1}\cdot\frac{d(\ln t)}{dt}\big) = (\ln t)^2 + 2t(\ln t)\frac{1}{t} = (\ln t)^2 + 2\ln t$. *Product Rule, then Power Rule.*

#19. $\frac{dy}{dt} = \frac{(\frac{d(\ln t}{dt}\cdot t) - (\ln t\cdot\frac{d(t)}{dt})}{t^2} = \frac{(\frac{1}{t})t - (\ln t)}{t^2} = \frac{1-\ln t}{t^2}$. *Quotient Rule.*

#21. $\frac{dy}{dx} = \frac{\big(\frac{d(\ln x)}{dx}(1+\ln x)\big) - \big((\ln x)(\frac{d(1+\ln x)}{dx}\big)}{(1+\ln x)^2} = \frac{\big((\frac{1}{x})(1+\ln x)\big) - \big((\ln x)(\frac{1}{x})\big)}{(1+\ln x)^2} = \frac{\frac{1}{x}}{(1+\ln x)^2} = \frac{1}{x(1+\ln x)^2}$. *Quotient Rule again.*

#25. $\frac{dy}{d\theta} = \frac{d(\theta)}{d\theta}\big(\sin(\ln\theta) + \cos(\ln\theta)\big) + \theta\Big(\frac{d\big(\sin(\ln\theta)+\cos(\ln\theta)\big)}{d\theta}\Big) = \big(\sin(\ln\theta) + \cos(\ln\theta)\big) + \theta\big(\frac{d(\sin(\ln\theta))}{d\theta} + \frac{d(\cos(\ln\theta))}{d\theta}\big) = \big(\sin(\ln\theta) + \cos(\ln\theta)\big) + \theta\big(\frac{d(\sin(\ln\theta))}{d\theta} + \frac{d(\cos(\ln\theta))}{d\theta}\big) = \big(\sin(\ln\theta) + \cos(\ln\theta)\big) + \theta\big(\cos(\ln\theta)\frac{d(\ln\theta)}{d\theta} - \sin(\ln\theta)\frac{d(\ln\theta)}{d\theta}\big) = \big(\sin(\ln\theta) + \cos(\ln\theta)\big) + \theta\big(\cos(\ln\theta)\frac{1}{\theta} + -\sin(\ln\theta)\frac{1}{\theta}\big) = 2\cos(\ln\theta)$. *Product Rule, Sum Rule, Chain Rule (twice).*

#37. Find $\frac{dy}{dx}$: $\ln y = \ln\sqrt{x(x+1)} = \frac{1}{2}\ln\big(x(x+1)\big) = \frac{1}{2}\big(\ln x + \ln(x+1)\big)$, so $\frac{\frac{dy}{dx}}{y} = \frac{1}{2}\big(\frac{1}{x} + \frac{1}{1+x}\big)$, so $\frac{dy}{dx} = \big(\sqrt{x(x+1)}\,\big)\big(\frac{1}{2}\big(\frac{1}{x} + \frac{1}{1+x}\big)\big)$.

#49. $\ln y = \ln\sqrt[3]{\frac{x(x-2)}{x^2+1}} = \frac{1}{3}\big(\ln x + \ln(x-2) - \ln(x^2+1)\big)$, so $\frac{\frac{dy}{dx}}{y} = \frac{1}{3}\big(\frac{1}{x} + \frac{1}{x-2} - \frac{\frac{d(x^2+1)}{dx}}{x^2+1}\big) = \frac{1}{3}\big(\frac{1}{x} + \frac{1}{x-2} - \frac{2x}{x^2+1}\big)$, so $\frac{dy}{dx} = \frac{1}{3}\sqrt[3]{\frac{x(x-2)}{x^2+1}}\Big(\frac{1}{x} + \frac{1}{x-2} - \frac{2x}{x^2+1}\Big)$.

#53. $\int\frac{2y\ dy}{y^2-25}$. Substitute $u = y^2 - 25$: $du = 2y\ dy$, so our integral becomes $\int\frac{du}{u} = \ln|u| + C = \ln|y^2 - 25| + C$.

#57. $\int_1^2 \frac{2\ln x}{x}dx$. Substitute $u = \ln x$: $du = \frac{1}{x}dx$. Hence our integral becomes $\int_{\ln 1}^{\ln 2} 2u\ du = u^2\Big|_0^{\ln 2} = (\ln 2)^2 - (0)^2 = (\ln 2)^2$. Note $(\ln 2)^2 \neq \ln 4$! It is true that $(\ln 2)^2 = \ln 2\cdot\ln 2 = \ln(2^{\ln 2})$.

#59. $\displaystyle\int_2^4 \frac{dx}{x(\ln x)^2}$. Substitute $u = \ln x$: $du = \frac{1}{x}\, dx$ so our integral becomes $\displaystyle\int_{\ln 2}^{\ln 4} \frac{du}{u^2} = -u^{-1}\Big|_{\ln 2}^{2\ln 2} = -\frac{1}{2\ln 2} + \frac{1}{\ln 2} = \frac{1}{2\ln 2}$. Note $2\ln 2 = \ln 4$.

#61. $\displaystyle\int \frac{3\sec^2 t}{6+3\tan t}\, dt$. Substitute $u = 6 + 3\tan t$: $du = 3\sec^2 t\; dt$ so our integral becomes $\displaystyle\int \frac{du}{u} = \ln|u| + C = \ln|6 + 3\tan t| + C$.