§6.4

#1. a) $5^{\log_5 7} = 7$ since $a^{\log_a x} = x$. b) $8^{\log_8 \sqrt{2}} = \sqrt{2}$ c) $1.3^{\log_{1.1} 75} = 75$ d) $\log_4 16 = \log_4(4^2) = 2$ since $\log_a(a^x) = x$ e) $\log_3 \sqrt{3} = \log_3 3^{1/2} = 1/2$ f) $\log_4(\frac{1}{4}) = \log_4 4^{-1} = -1$.

#5. a) $\frac{\log_2 x}{\log_3 x} = \frac{\frac{\ln x}{\ln 2}}{\frac{\ln x}{\ln 3}} = \frac{\ln 3}{\ln 2}$. b) $\frac{\log_2 x}{\log_8 x} = \frac{\frac{\ln x}{\ln 2}}{\frac{\ln x}{\ln 8}} = \frac{\ln 8}{\ln 2} = \frac{\ln 2^3}{\ln 2} = \frac{3\ln 2}{\ln 2} = 3$ c) $\frac{\log_x a}{\log_{x^2} a} = \frac{\frac{\ln a}{\ln x}}{\frac{\ln a}{\ln x^2}} = \frac{\ln x^2}{\ln x} = \frac{2\ln x}{\ln x} = 2$.

#9. Solve $3^{\log_3(x^2)} = 5e^{\ln x} - 3\cdot 10^{\log_{10}(2)}$. This simplifies to $x^2 = 5x - 3\cdot 2$ or $x^2 - 5x + 6 = 0$. Factoring we see $x = 2$ or $x = 3$.

#13. Find $\frac{dy}{ds}$ where $y = 5^{\sqrt{s}}$. Chain rule: $\frac{dy}{ds} = 5^{\sqrt{s}}(\ln 5)\frac{d\sqrt{s}}{ds} = (\ln 5)5^{\sqrt{s}}(\frac{1}{2}s^{-1/2}) = \frac{\ln 5}{2}\frac{5^{\sqrt{s}}}{\sqrt{s}}$.

#17. Find $\frac{dy}{d\theta}$ where $y = (\cos\theta)^{\sqrt{2}}$. Power rule: $\frac{dy}{d\theta} = (\sqrt{2})(\cos\theta)^{\sqrt{2}-1}\frac{d\cos\theta}{d\theta} = (\sqrt{2})(\cos\theta)^{\sqrt{2}-1}(-\sin\theta)$.

#33. Find $\frac{dy}{dx}$ where $y = \log_5 e^x$. Chain rule: $\frac{dy}{dx} = \frac{1}{\ln 5}\frac{1}{e^x}\frac{de^x}{dx} = \frac{1}{\ln 5}\frac{1}{e^x}e^x = \frac{1}{\ln 5}$. OR, simplify: $\log_5 e^x = \frac{\ln e^x}{\ln 5} = \frac{x}{\ln 5}$ and the derivative is now easy.

#39. Find $y' = \frac{dy}{dx}$ if $y = (x+1)^x$. Take log of both sides and simplify. $\ln y = \ln\big((x+1)^x\big) = x\ln(x+1)$. Now take the derivative of both sides. $\frac{y'}{y} = \frac{dx}{dx}\ln(x+1) + x\frac{d\ln(x+1)}{dx} = \ln(x+1) + \frac{\frac{dx+1}{dx}}{x+1} = \ln(x+1) + \frac{1}{x+1}$. Hence $y' = (x+1)^x\big(\ln(x+1) + \frac{x}{x+1}\big)$.

#43. Find $y' = \frac{dy}{dx}$ if $y = (\sin x)^x$. Take log of both sides and simplify. $\ln y = \ln\big((\sin x)^x\big) = x\ln(\sin x)$. Now take the derivative of both sides. $\frac{y'}{y} = \frac{dx}{dx}\ln(\sin x) + x\frac{d\ln(\sin x)}{dx}$. Compute $\frac{d\ln(\sin x)}{dx} = \frac{\frac{d\sin x}{dx}}{\sin x} = \frac{-\cos x}{\sin x} = -\cot x$. Continuing, $\frac{y'}{y} = \ln(\sin x) - x\cot x$, so $y' = (\sin x)^x\big(\ln(\sin x) - x\cot x\big)$.

#47. $\displaystyle\int 5^x\,dx = \frac{5^x}{\ln 5} + C$ from formula.

#51. $\displaystyle\int_1 \sqrt{2}x2^{x^2}dx$. Substitute $u = x^2$: then $du = 2xdx$ so $\displaystyle\int_1 \sqrt{2}x2^{x^2}dx = \int_1^2 \frac{1}{2}2^u du = \frac{1}{2}(\frac{2^u}{\ln 2})\Big|_1^2 = \frac{1}{2}(\frac{2^2}{\ln 2} - \frac{2^1}{\ln 2}) = \frac{1}{2}(\frac{4}{\ln 2} - \frac{2}{\ln 2}) = \frac{1}{2}\frac{2}{\ln 2} = \frac{1}{\ln 2}$.

#57. $\displaystyle\int 3x^{\sqrt{3}}dx$. You knew how to do this one long ago: $\displaystyle\int 3x^{\sqrt{3}}dx = 3\frac{x^{\sqrt{3}+1}}{\sqrt{3}+1} + C$.

#65. $\displaystyle\int_0^2 \frac{\log_2(x+2)}{x+2}dx$. Substitute $u = \log_2(x+2)$ so $du = \frac{1}{\ln 2}\frac{\frac{dx+2}{dx}}{x+2}dx = \frac{1}{\ln 2}\frac{1}{x+2}dx$ so $\displaystyle\int_0^2 \frac{\log_2(x+2)}{x+2}dx = \int_1^2 (\ln 2)udu$. The limits follow since $\log_2(0+2) = 1$ and $\log_2(2+2) = \log_2(4) = \log_2(2^2) = 2$. Continuing, $\displaystyle\int_1^2 (\ln 2)u\,du = (\ln 2)\frac{u^2}{2}\Big|_1^2 = (\ln 2)(\frac{2^2}{2} - \frac{1^2}{2}) =$

$(\ln 2)(2 - \frac{1}{2}) = \frac{3}{2}\ln 2.$