§6.5

#3. The differential equation is $\frac{dy}{dt} = -0.6y$ and the initial value condition is $y(0) = 100$ provided we measure time starting from now (and recall that it is already being measured in hours) and y denotes the number of grams of δ–glucono lactone acid. The problem asks for $y(1)$. Solve by finding $y(t)$ and then plug in $t = 1$. $y = y_0 e^{kt}$, where $k = -0.6$ and $y_0 = 100$. Hence $y(1) = y_0 e^{k\cdot 1} = 100e^{-0.6}$. Without a calculator, this is an acceptable form for the answer.

#5. Here $L(x)$ is measuring intensity of light in some unspecified units. The governing equation is $\frac{dL}{dx} = -kL$ where k depends on the water into which you are diving. You are given $L(18) = \frac{1}{2}L(0)$, and you are asked to find x so that $L(x) = \frac{1}{10}L(0)$. As usual, $L = L(0)e^{kx}$. From the equation $L(18) = \frac{1}{2}L(0)$ we find $L(0)e^{18k} = \frac{1}{2}L(0)$, so $e^{18k} = \frac{1}{2}$. Hence $L = L(0)(\frac{1}{2})^{\frac{x}{18}}$. Hence we need to solve $L(0)(\frac{1}{2})^{\frac{x}{18}} = \frac{1}{10}L(0)$, or $(\frac{1}{2})^{\frac{x}{18}} = \frac{1}{10}$, or $2^{\frac{x}{18}} = 10$, or $\frac{x}{18}\ \ln 2 = \ln 10$, so $x = 18\frac{\ln 10}{\ln 2}$.

#7. Here the governing equation is $\frac{dB}{dt} = kB$ where B is the number of bacteria and t is the time measured in hours from now. The initial condition is $B(0) = 1$ and we are given the additional information that $B(0.5) = 2B(0) = 2$. We are asked for $B(24)$. Now $B = B(0)e^{kt} = e^{kt}$ so our additional information gives us $B(0.5) = e^{0.5k} = 2$. Hence $B = 2^{2t} = 4^t$ and $B(24) = 2^{48}$ or 4^{24}.

#13. The differential equation is $A = A_0 e^{rt}$ where r is the rate of interest, 4%. a) In 5 years you will have $A = A_0 e^{0.04\cdot 5} = A_0 e^{0.2}$. b) The doubling time is the time t which satisfies the equation $A(t) = 2A_0$. Hence $A_0 e^{0.04t} = 2A_0$, so $e^{0.04t} = 2$ or $0.04t = \ln 2$ or $t = \frac{\ln 2}{0.04}$. The tripling time is given by the equation $A(t) = 3A_0$ and is solved the same way: $t = \frac{\ln 3}{0.04}$.