## §6.7

#1. Any power of $x$ grows more slowly that $a^x$ for any $a > 1$. For any $a > 1$, $\log_a x$ grows more slowly than any power of $x$. Hence a) and c) grow more slowly than $e^x$; $\log_{10} x$ grows more slowly than $x$ and hence more slowly than $e^x$. Since $\sin x$ is bounded $x^2$ and $x^2 + \sin^2 x$ grow at the same rate. Hence $x^2 + \sin^2 x$ grows more slowly than $e^x$. Both $e^x$ and $e^x/2$ grow at the same rate. Finally, if$a > b > 1$ $a^x$ grows faster than $b^x$. Since $3/2 < e$, $(3/2)^x$ grows more slowly than $e^x$. Since $4 > e$, $4^x$ grows faster than $e^x$. Since $\sqrt{e} < e$, $e^{x/2}$ grows more slowly than $e^x$.

#3. $x^2 + 4x$, $\sqrt{x^4 x^3}$, $(x+3)^2$, $8x^2$ and $x^2$ all grow at the same rate. $x^5 - x^2$ grows faster than $x^2$. $2^x$ grows faster than any power of $x$, hence faster than $x^2$. $x^3 e^{-x}$ goes to 0 as $x \to \infty$ so it geows more slowly than $x^2$. Finally, $x$ grows faster than $\ln x$ so $x^2$ grows faster than $x \ln x$.

#5. $\ln x$ and $\log_a x$ grow at the same rate. Hence $\log_3 x$, $\ln 2x = \ln 2 + \ln x$, $\ln \sqrt{x} = \frac{1}{2} \ln x$, $5 \ln x$ and $\ln x$ all grow at the same rate. $\ln x$ grow more slowly that $x$ and $\sqrt{x} = x^{1/2}$. $\ln x$ grows more slowly than $e^x$. Finally $\frac{1}{x}$ goes to 0 as $x \to \infty$ and so grows more slowly than $\ln x$.

#7. $e^x$ and $e^{x/2}$ grow at the same rate; since $e < 4$, $e^x$ grows more slowly than $4^x$ which is less than both $x^x$ and $(\ln x)^x$, so $e^x$ grows more slowly than $(\ln x)^x$. Finally, $(\ln x)^x$ grows more slowly than $x^x$ since $\lim\limits_{\to\infty} x \dfrac{x^x}{(\ln x)^x} = \infty$.

#9. a) $\lim\limits_{x\to\infty} \dfrac{x}{x} = 1$ so false, b) $\lim\limits_{x\to\infty} \dfrac{x}{x+5} = 1$ so false, c) $\frac{x}{x+5} < 1$ for large $x$ so true, d) $\frac{x}{2x} = \frac{1}{2}$, so true, e) $\lim\limits_{x\to\infty} \dfrac{e^x}{e^{2x}} = \lim\limits_{x\to\infty} \dfrac{1}{e^x} = 0$ so true, f) $\frac{x+\ln x}{x}$: since $\lim\limits_{x\to\infty} \dfrac{\ln x}{x} = 0$, for all large $x$, $\frac{\ln x}{x} < 1$ so $\frac{x+\ln x}{x} < 2$ for all large $x$, so $x + \ln x = O(x)$. g) $\lim\limits_{x\to\infty} \dfrac{\ln x}{\ln(2x)} = 1$ so $\ln x \neq o(\ln(2x))$. h) $\lim\limits_{x\to\infty} \dfrac{\sqrt{x^2+5}}{x} = 1$ so for all sufficiently large $x$, $\frac{\sqrt{x^2+5}}{x} < 2$, so $\sqrt{x^2+5} = O(x)$.

#23.a) $\log_2 n$ grows more slowly than any power of $n$, so $n \log_2 n$ grows more slowly than $n^{3/2}$. Similarly, one can see $n(\log_2 n)^2$ grow more slowly than $n^{3/2}$. Finally, $\log_2 n$ grow more slowly than $(\log_2 n)^2$, so $n \log_2 n$ grows the most slowly of the three given functions.