§6.8

Useful angles:

$\cos 0 = 1$, $\sin 0 = 0$
$\cos\frac{\pi}{2} = 0$, $\sin\frac{\pi}{2} = 1$
$\cos\pi = -1$, $\sin 0\pi = 0$
$\cos(\frac{\pi}{6}) = \frac{1}{2}$, $\sin(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$
$\cos(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$, $\sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$
$\cos(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$, $\sin(\frac{\pi}{3}) = \frac{1}{2}$

#2. a) $\arctan(-1) = -\frac{\pi}{4}$ since $\tan(\frac{-\pi}{4}) = -1$ and $-\frac{\pi}{2} < -\frac{\pi}{4} < \frac{\pi}{2}$.
b) $\arctan\sqrt{3} = \frac{\pi}{6}$ since $\tan(\frac{\pi}{6}) = \sqrt{3}$
c) $\arctan\frac{-1}{\sqrt{3}} = -\frac{\pi}{3}$ since $\tan(-\frac{\pi}{3}) = -\frac{1}{\sqrt{3}}$

#19. $\tan\Big(\arcsin(-\frac{1}{2})\Big)$ =? But $\arcsin(-\frac{1}{2}) = -\frac{\pi}{3}$ since $\sin(-frac\pi 3) = -frac12$, so $\tan\Big(\arcsin(-\frac{1}{2})\Big) = \tan(-\frac{\pi}{3}) = -\tan(\frac{\pi}{3}) = \frac{-1}{\sqrt{3}}$.

#31. $\tan\big(\operatorname{arcsec}(3y)\big)$ =?. Set up a right triangle with one angle θ with $\sec\theta = 3y$. Then $(3y)^2 = \tan^2(3y) + 1$, so $\tan(3y) = \sqrt{9y^2 - 1}$.

#41. $\lim\limits_{x\to 1^-} \arcsin x = \dfrac{\pi}{2}$ since the inverse of a continuous function is continuous and $\lim\limits_{x\to\frac{\pi}{2}^-} \sin x = 1$.

#43. $\lim\limits_{x\to\infty} \arctan x = \dfrac{\pi}{2}$ since $\lim\limits_{x\to\frac{\pi}{2}^-} \tan x = \infty$.

#49. Let β be the angle with sides x and 3 from the picture on page 511 in the book. Then $\cot\beta = \frac{x}{3}$ and $\cot(\alpha+\beta) = \frac{x}{3+12} = \frac{x}{15}$ so $\alpha+\beta = \operatorname{arccot}(\frac{x}{15})$ and $\beta = \operatorname{arccot}(\frac{x}{3})$ so $\alpha = \operatorname{arccot}(\frac{x}{15}) - \operatorname{arccot}(\frac{x}{3})$.