§6.9

#1. $\frac{d \arccos x^2}{dx} = \frac{\frac{dx^2}{dx}}{\sqrt{1-(x^2)^2}} = \frac{2x}{\sqrt{1-x^4}}$.

#3. $\frac{d \arcsin \sqrt{2}\, t}{dt} = \frac{1}{\sqrt{1-(\sqrt{2}\, t)^2}} \frac{d\sqrt{2}\, t}{dt} = \frac{\sqrt{2}}{\sqrt{1-2t^2}}$.

#5. $\frac{d\text{arcsec}\,(2s+1)}{ds} = \frac{1}{|2s+1|\sqrt{(2s+1)^2-1}} \frac{d2s+1}{ds} = \frac{2}{|2s+1|\sqrt{4s^2+4s}} = \frac{1}{|2s+1|\sqrt{s^2+s}}$.

#8. $\frac{d\text{arccsc}\,(\frac{x}{2})}{dx} = -\frac{1}{|\frac{x}{2}|\sqrt{(\frac{x}{2})^2-1}} \frac{d\frac{x}{2}}{dx} = -\frac{\frac{1}{2}}{|\frac{x}{2}|\sqrt{\frac{x^2}{4}-1}} = \frac{-8}{|x|\sqrt{x^2-4}}$.

#17. $\frac{ds\sqrt{1-s^2}+\arccos s}{ds} = 1 \cdot \sqrt{1-s^2} + s(-2s)\frac{1}{2}(1-s^2)^{-1/2} - \frac{1}{\sqrt{1-s^2}} =$
$\frac{1-s^2}{\sqrt{1-s^2}} + \frac{-s^2}{\sqrt{1-s^2}} - \frac{1}{\sqrt{1-s^2}} = \frac{-2s^2}{\sqrt{1-s^2}}$.

#23. $\int \frac{dx}{\sqrt{9-x^2}}$. Substitute $3u = x$; $3du = dx$, so $\int \frac{dx}{\sqrt{9-x^2}} = \int \frac{3du}{\sqrt{9-9u^2}} = \int \frac{du}{\sqrt{1-u^2}} =$
$\arcsin u + C = \arcsin(\frac{x}{3}) + C$.

#25. $\int \frac{dx}{17+x^2}$. Substitute $\sqrt{17}\, u = x$ so $\sqrt{17}\, du = dx$ and $\int \frac{dx}{17+x^2} = \int \frac{\sqrt{17}\, du}{17+17u^2} =$
$\int \frac{\sqrt{du}}{1+1u^2} = \arctan u + C = \arctan(\frac{x}{\sqrt{17}}) + C$.

#27. $\int \frac{dx}{x\sqrt{25x^2-2}}$. Substitute $\sqrt{2}\, u = 5x$; $\sqrt{2}\, du = 5dx$. Hence $\int \frac{dx}{x\sqrt{25x^2-2}} =$
$\int \frac{\frac{\sqrt{2}\, du}{5}}{(\frac{\sqrt{2}\, u}{5})\sqrt{2u^2-2}} = \frac{1}{\sqrt{2}} \int \frac{du}{u\sqrt{u^2-1}} = \begin{cases} \frac{1}{\sqrt{2}}\text{arcsec}\, u + C & \text{if } u > 1 \\ \frac{-1}{\sqrt{2}}\text{arcsec}\, u + C & \text{if } u < 1 \end{cases} =$
$\frac{1}{\sqrt{2}}\text{arcsec}\, |u| + C = \frac{1}{\sqrt{2}}\text{arcsec}\left(\left|\frac{\sqrt{2}\, x}{5}\right|\right) + C$.

#29. $\int_0^1 \frac{4ds}{\sqrt{4-s^2}}$. Substitute $2u = s$, $(2du = ds)$ so $\int_0^1 \frac{4ds}{\sqrt{4-s^2}} = \int_0^{\frac{1}{2}} \frac{8du}{\sqrt{4-4u^2}} =$
$4\int_0^{\frac{1}{2}} \frac{du}{\sqrt{1-u^2}} = 4\arcsin(u)\Big|_0^{1/2} = 4(\frac{\pi}{6} - 0) = \frac{2\pi}{3}$.

#31. $\int_0^2 \frac{dt}{8+2t^2}$. Substitute $2u = t$ $(2du = dt)$ so $\int_0^2 \frac{dt}{8+2t^2} = \int_0^1 \frac{2du}{8+8t^2} = \frac{1}{4}\int_0^1 \frac{du}{1+t^2} =$
$\frac{\arctan u}{4}\Big|_0^1 = \frac{\arctan 1 - \arctan 0}{4} = \frac{\frac{\pi}{4} - 0}{4} = \frac{\pi}{16}$.

#33. $\int_{-1}^{-\frac{\sqrt{2}}{2}} \frac{dy}{y\sqrt{4y^2-1}}$. Substitute $u = 2y$ $(du = 2dy)$. $\int_{-1}^{-\frac{\sqrt{2}}{2}} \frac{dy}{y\sqrt{4y^2-1}} = \int_{-2}^{-\sqrt{2}} \frac{\frac{du}{2}}{\frac{u}{2}\sqrt{u^2-1}}$
$\int_{-2}^{-\sqrt{2}} \frac{du}{u\sqrt{u^2-1}} = \text{arcsec}\, |u|\Big|_{-2}^{-\sqrt{2}} = \text{arcsec}\, \sqrt{2} - \text{arcsec}\, 2 = \frac{\pi}{4} - \frac{\pi}{3} = -\frac{\pi}{12}$.

#35. $\int \frac{3dr}{\sqrt{1-4(r-1)^2}}$. Substitute $u = 2(r-1)$ $(du = 2dr)$. $\int \frac{3dr}{\sqrt{1-4(r-1)^2}} = \int \frac{\frac{3}{2}du}{\sqrt{1u^2}} = \frac{3}{2}\arcsin u + C = \frac{3}{2}\arcsin\Big(2(r-1)\Big) + C$.

#37. $\int \frac{dx}{2+(x-1)^2}$. Substitute $\sqrt{2}\,u = (x-1)$ $(\sqrt{2}\,du = dx$. $\int \frac{dx}{2+(x-1)^2} = \int \frac{\sqrt{2}\,du}{2+2u^2} = \frac{\sqrt{2}}{2}\int \frac{du}{1+u^2} = \frac{\sqrt{2}}{2}\arctan u + C = \frac{\sqrt{2}}{2}\arctan\Big(\frac{x-1}{\sqrt{2}}\Big) + C$.

#39. $\int \frac{dx}{(2x-1)\sqrt{(2x-1)^2-4}}$. Substitute $2u = 2x-1$ $(2du = 2dx$ or $du = dx)$. $\int \frac{dx}{(2x-1)\sqrt{(2x-1)^2-4}} = \int \frac{du}{(2u)\sqrt{(2u)^2-4}} = \frac{1}{2\cdot 2}\int \frac{du}{u\sqrt{(u)^2-1}} = \frac{1}{4}\int \operatorname{arcsec}|u| + C = \frac{1}{4}\int \operatorname{arcsec}\left|\frac{2x-1}{2}\right| + C$.

#41. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{2\cos\theta\, d\theta}{1+(\sin\theta)^2}$. Substitute $u = \sin\theta$ $(du = \cos\theta\, d\theta)$. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{2\cos\theta\, d\theta}{1+(\sin\theta)^2} = \int_{-1}^{1} \frac{2du}{1+u^2} = 2\arctan u\Big|_{-1}^{1} = 2\Big(\frac{\pi}{4} - (-\frac{\pi}{4})\Big) = 2\cdot\frac{\pi}{2} = \pi$.

#43. $\int_0^{\ln\sqrt{3}} \frac{e^x\,dx}{1+e^{2x}}$. Substitute $u = e^x$ $(du = e^x dx)$. $\int_0^{\ln\sqrt{3}} \frac{e^x\,dx}{1+e^{2x}} = \int_1^{\sqrt{3}} \frac{du}{1+u^2} = \arctan u\Big|_1^{sqrt3} = \arctan\sqrt{3} - \arctan 1 = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$.

#45. $\int \frac{ydy}{\sqrt{1-y^4}}$. Substitute $u = y^2$ $(du = 2ydy)$. $\int \frac{ydy}{\sqrt{1-y^2}} = \int \frac{\frac{du}{2}}{\sqrt{1-u^2}} == \frac{1}{2}\arcsin u + C = \frac{1}{2}\arcsin(y^2) + C$.

#47. $\int \frac{dx}{\sqrt{-x^2+4x-3}}$. Complete the square: $-x^2+4x-3 = -(x-2)^2+1$, so substitute $u = x-2$ $(du = dx)$. $\int \frac{dx}{\sqrt{-x^2+4x-3}} = \int \frac{du}{\sqrt{1-u^2}} = \arcsin u + C = \arcsin(x-2) + C$.

#49. $\int_{-1}^{0} \frac{6dt}{\sqrt{3-2t-t^2}}$. Complete the square: $3-2t-t^2 = -(t+1)^2+4$ so substitute $2u = t+1$ $(2du = dt)$. $\int_{-1}^{0} \frac{6dt}{\sqrt{3-2t-t^2}} = \int_0^{\frac{1}{2}} \frac{12du}{\sqrt{4-4u^2}} = \frac{12}{2}\int_0^{\frac{1}{2}} \frac{du}{\sqrt{1-u^2}} = 6\arcsin u\Big|_0^{\frac{1}{2}} = 6(\frac{\pi}{6} - 0) = \pi$.

#51. $\int \frac{dy}{y^2-2y+5}$. Complete the square: $y^2-2y+5 = (y-1)^2+4$ so substitute $2u = y-1$ $(2du = dy)$. $\int \frac{dy}{y^2-2y+5} = \int \frac{2du}{4u^2+4} = \frac{2}{4}\int \frac{du}{1+u^2} = \frac{1}{2}\arctan u + C = \frac{1}{2}\arctan\Big(\frac{y-1}{2}\Big) + C$.

#53. $\int_1^2 \frac{8dx}{x^2-2x+2}$. Complete the square: $x^2-2x+2 = (x-1)^2+1$ so substitute $u = x-1$ $(du = dx)$. $\int_1^2 \frac{8dx}{x^2-2x+2} = \int_0^1 \frac{8du}{u^2+1} = 8\arctan u\Big|_0^1 = 8(\arctan 1 - \arctan 0) = 8(\frac{\pi}{4} - 0) = 2\pi$.

#55. $\int \frac{dx}{(x+1)\sqrt{x^2+2x}}$. Complete the square: $x^2+2x=(x+1)^2-1$ so substitute $u=x+1$ $(du=dx)$. $\int \frac{dx}{(x+1)\sqrt{x^2+2x}} = \int \frac{du}{u\sqrt{u^2-1}} = \operatorname{arcsec}|u| + C = \operatorname{arcsec}|x+1| + C$.

#57. $\int \frac{e^{\arcsin x}dx}{\sqrt{1-x^2}}$. Substitute $u=\arcsin x$ $(du=\frac{dx}{\sqrt{1-x^2}})$. $\int \frac{e^{\arcsin x}dx}{\sqrt{1-x^2}} = \int e^u du = e^u + C = e^{\arcsin x} + C$.

#59. $\int \frac{(\arcsin x)^2 dx}{\sqrt{1-x^2}}$. Substitute $u=\arcsin x$ $(du=\frac{dx}{\sqrt{1-x^2}})$. $\int \frac{(\arcsin x)^2 dx}{\sqrt{1-x^2}} = \int u^2 du = \frac{u^3}{3} + C = \frac{(\arcsin x)^3}{3} + C$.

#61. $\int \frac{dy}{(\arctan y)(1+y^2)}$. Substitute $u=\arctan y$ $(du=\frac{dy}{1+y^2})$. $\int \frac{dy}{(\arctan y)(1+y^2)} = \int \frac{du}{u} = \ln|u| + C = \ln|\arctan x| + C$.

#63. $\int_{\sqrt{2}}^{2} \frac{\sec^2(\operatorname{arcsec} x)dx}{x\sqrt{x^2-1}}$. First simplify: $\sec(\operatorname{arcsec} x) = x$ so $\int_{\sqrt{2}}^{2} \frac{\sec^2(\operatorname{arcsec} x)dx}{x\sqrt{x^2-1}} = \int_{\sqrt{2}}^{2} \frac{x^2 dx}{x\sqrt{x^2-1}} = \int_{\sqrt{2}}^{2} \frac{x dx}{\sqrt{x^2-1}}$. Now substitute $u=x^2-1$ $(du=2x\,dx)$. $\int_{\sqrt{2}}^{2} \frac{\sec^2(\operatorname{arcsec} x)dx}{x\sqrt{x^2-1}} = \int_1^3 \frac{1}{2}\frac{du}{\sqrt{u}} = \int_1^3 \frac{1}{2}u^{-1/2}du = \frac{1}{2}\frac{u^{-1/2+1}}{-1/2+1}\Big|_1^3 = \frac{1}{2}\frac{u^{1/2}}{1/2}\Big|_1^3 = u^{1/2}\Big|_1^3 = \sqrt{3}-\sqrt{1} = \sqrt{3}-1$.

#65. $\lim_{x\to 0} \frac{\arcsin 5x}{x}$. Since $\arcsin(5\cdot 0) = \arcsin(0) = 0$ this limit has the form $\frac{0}{0}$. By l'Hôpital's Rule, $\lim_{x\to 0} \frac{\arcsin 5x}{x} = \lim_{x\to 0} \frac{\frac{d \arcsin 5x}{dx}}{1} = \lim_{x\to 0} \frac{\frac{\frac{d5x}{dx}}{\sqrt{1-(5x)^2}}}{1} = \lim_{x\to 0} \frac{5}{\sqrt{1-25x^2}} = \frac{5}{\sqrt{1-25\cdot 0^2}} = 5$.

#73. Solve $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$, $y(0)=0$. First solve the differential equation: $y = \arcsin x + C$. Then solve $0 = y(0) = \arcsin(0) + C$. Since $\arcsin(0) = 0$, $C=0$ and the solution is $y = \arcsin x$.