§7.01

#3. $\int 3\sqrt{\sin\ v}\ \cos v\, dv$: substitute $w = \sin v$, $dw = \cos v\, dv$ so $\int 3\sqrt{w}\, dw = 3\int w^{1/2}\, dw =$ $3\frac{w^{1+1/2}}{1+1/2} + C = 2\ w^{\frac{3}{2}} + C = 2(\sin\ v)^{\frac{3}{2}} + C$.

#7. $\int \frac{dx}{\sqrt{x}(\sqrt{x}+1)}$: substitute $u = \sqrt{x}+1$, $du = d(x^{1/2}) = \frac{1}{2}x^{\frac{1}{2}-1}dx = \frac{1}{2\sqrt{x}}dx$ so $\int \frac{dx}{\sqrt{x}(\sqrt{x}+1)} =$ $\int \frac{2}{u}\ du = 2\ln|u| + C = 2\ln(\sqrt{x}+1) + C$. The absolute value signs would be correct also, but they can be dropped since $\sqrt{x}+1 > 0$

#11. $\int e^{\theta}\csc(e^{\theta}+1)d\theta$: substitute $u = e^{\theta}+1$, $du = e^{\theta}d\theta$. $\int e^{\theta}\csc(e^{\theta}+1)d\theta = \int \csc u\, du =$ $-\ln\left|\csc\ u + \cot\ u\right| + C = -\ln\left|\csc(e^{\theta}+1) + \cot(e^{\theta}+1)\right| + C$.

#17. $\int_0^{\sqrt{\ln 2}} 2xe^{x^2}\ dx$: substitute $u = x^2$, $du = 2x\ dx$ so $\int_0^{\sqrt{\ln 2}} 2xe^{x^2}\ dx = \int_0^{\ln 2} e^u\ du =$ $e^u\Big|_0^{\ln 2} = e^{\ln 2} - e^0 = 2 - 1 = 1$.

#31. $\int \frac{6dx}{x\sqrt{25x^2-1}}$: substitute $u = 5x$, $du = 5dx$ or $\frac{6}{5}du = 6dx$ so $\int \frac{6dx}{x\sqrt{25x^2-1}} = \int \frac{\frac{6}{5}du}{\frac{u}{5}\sqrt{u^2-1}} =$ $6\int \frac{du}{u\sqrt{u^2-1}} = 6\mathrm{arc\,sec}\,|u| + C = 6\mathrm{arc\,sec}\,|5x| + C$.

#35. $\int_1^{e^{\pi/3}} \frac{dx}{x\cos(\ln\ x)}$: substitute $u = \ln x$, $du = \frac{dx}{x}$. $\int_1^{e^{\pi/3}} \frac{dx}{x\cos(\ln\ x)} = \int_0^{\pi/3} \frac{du}{\cos(u)} =$ $\int_0^{\pi/3} \sec u\, du = \ln|\sec u + \tan u|\Big|_0^{\pi/3} = \ln\big(\sec(\pi/3)+\tan(\pi/3)\big) - \ln\big(\sec(0)+\tan(0)\big) =$ $\ln\big(2+\sqrt{3}\big) - \ln\big(1+0\big) = \ln(2+\sqrt{3})$

#41. $\int \frac{dx}{(x+1)\sqrt{x^2+2x}}$. Completing the square: $x^2 + 2x = (x+1)^2 - 1$. Substitute $u = x+1$, $du = dx$, so $\int \frac{dx}{(x+1)\sqrt{x^2+2x}} = \int \frac{du}{u\sqrt{u^2-1}} = \mathrm{arc\,sec}\,|u| + C = \mathrm{arc\,sec}\,|x+1| + C$.

#47. $\int \frac{x\ dx}{x+1}$. Polynomial long division $\frac{x}{x+1} = 1 - \frac{1}{x+1}$ so $\int \frac{x\ dx}{x+1} = \int \left(1 - \frac{1}{x+1}\right)\ dx =$ $x - \ln|x+1| + C$.

#49. $\int \frac{2x^3}{x^2-1}dx$. Polynomial long division yields $\frac{2x^3}{x^2-1} = 2x + \frac{2x}{x^2-1}$ so $\int \frac{2x^3}{x^2-1}dx = \int 2xdx +$ $\int \frac{2x}{x^2-1}\ dx = x^2 + \ln|x^2-1| + C$.

#53. $\int \frac{1-x}{\sqrt{1-x^2}}\, dx = \int \frac{1}{\sqrt{1-x^2}}\, dx - \int \frac{x}{\sqrt{1-x^2}}\, dx$. Do the integral $\int \frac{x}{\sqrt{1-x^2}}\, dx$ by substitution $u = 1 - x^2$, $du = -2x\, dx$, so $\int \frac{x}{\sqrt{1-x^2}}\, dx = -\frac{1}{2}\int \frac{du}{\sqrt{u}} = -\frac{1}{2}\int u^{-1/2}du = -\frac{1}{2}\frac{u^{1-1/2}}{1-1/2} +$

$C = -\frac{1}{2}\frac{u^{1/2}}{1/2} + C = -\sqrt{u} + C = -\sqrt{1-x^2} + C$. Hence $\int \frac{1}{\sqrt{1-x^2}}\,dx - \int \frac{x}{\sqrt{1-x^2}}\,dx =$ $\arcsin x - (-\sqrt{1-x^2}) + C = \arcsin x + \sqrt{1-x^2} + C$.

#57. $\int \frac{1}{1+\sin x} dx = \int \frac{1}{1+\sin x}\frac{1-\sin x}{1-\sin x} dx = \int \frac{1-\sin x}{1-\sin^2 x} dx = \int \frac{1-\sin x}{\cos^2 x} dx = \int \sec^2 x\ dx - \int \sec x \tan x\ dx = \tan x - \sec x + C$.

#65. $\int_{\pi/2}^{\pi} \sqrt{1+\cos 2t}\,dt$. The Double Angle Formula yields $2\cos^2 t = 1+\cos 2t$, so $\int_{\pi/2}^{\pi} \sqrt{1+\cos 2t}\,dt = \int_{\pi/2}^{\pi} \sqrt{2\cos^2 t}\,dt = \sqrt{2}\int_{\pi/2}^{\pi} |\cos t| dt = -\sqrt{2}\int_{\pi/2}^{\pi} \cos t dt = -\sqrt{2}\sin t\Big|_{\pi/2}^{\pi} = -\sqrt{2}\big(\sin\pi - \sin(\pi/2)\big) = -\sqrt{2}(0-1) = \sqrt{2}$