§9.09

1. Area inside the oval limaçon $r = 4 + 2\cos\theta$. To graph, start with $\theta = 0$ so $r = 6$. Compute $\frac{dr}{d\theta} = -2\sin\theta$. Interesting points are where $\frac{dr}{d\theta}$ vanishes, or at $\theta = 0$, π, 2π, etc. For these values of θ we compute r: $(6, 0)$, $(2, \pi)$ and the values repeat. Hence, starting at $\theta = 0$ and rotating counterclockwise, we see the point moving in along the ray starting at 6 until at $\theta = \pi$ is has moved into 2. As the ray moves from $\theta = \pi$ to $\theta = 2\pi$, the point move out along the ray starting a 2 and finishing at 6. Just to get a good picture it is worthwhile to plug in $\theta = \frac{\pi}{2}$ and $\theta = \frac{3\pi}{2}$ where $r = 4$. Hence the area we want is swept out once as θ rotates from 0 to 2π.

From the formula in the book **Area** $= \frac{1}{2}\int_0^{2\pi} r^2\, d\theta = \frac{1}{2}\int_0^{2\pi} (4 + 2\cos\theta)^2)d\theta = \frac{1}{2}\Big(16\int_0^{2\pi} d\theta + 16\int_0^{2\pi} \cos\theta\, d\theta + 4\int_0^{2\pi} \cos^2\theta\, d\theta\Big)$. Do the pieces: $\int_0^{2\pi} d\theta = \theta\Big|_0^{2\pi} = 2\pi$; $\int_0^{2\pi} \cos\theta\, d\theta = \sin\theta\Big|_0^{2\pi} = 0 - 0 = 0$; $\int_0^{2\pi} \cos^2\theta\, d\theta = \int_0^{2\pi} \frac{1 + \cos(2\theta)}{2} d\theta = \frac{1}{2}\int_0^{2\pi} d\theta + \frac{1}{2}\int_0^{2\pi} \cos(2\theta) d\theta$. Pause to do $\int_0^{2\pi} \cos(2\theta) d\theta = \frac{1}{2}\sin(2\theta)\Big|_0^{2\pi} = 0 - 0 = 0$. Hence $\int_0^{2\pi} \cos^2\theta d\theta = \frac{1}{2} \cdot 2\pi = \pi$. Hence the **Area** is $\frac{1}{2}(16 \cdot 2\pi + 16 \cdot 0 + 4 \cdot \pi) = 18\pi$.

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3. Area inside one leaf of the four-leafed rose $r = \cos(2\theta)$. Begin with the graph, starting with $\theta = 0$. $\frac{dr}{d\theta} = -2\sin(2\theta)$ which vanishes when $2\theta = 0$, π, 2π, 3π, 4π, etc. or when $\theta = 0$, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, 2π, etc.: r itself vanishes when $\theta = \frac{\pi}{4}$, $\frac{3\pi}{4}$, $\frac{5\pi}{4}$, $\frac{7\pi}{4}$, etc.

A remark which is apparent if **you** draw the graph but not if you just look at it is that from 0 to $\frac{\pi}{4}$ you trace out the top half of the right-hand leaf, **but** from $\frac{\pi}{4}$ to $\frac{\pi}{2}$ you trace out the left-half of the lower leaf. You have many choices for a range of θ which sweep out one leaf: $[\frac{\pi}{4}$ to $\frac{3\pi}{4}]$ sweeps out the lower leaf; $[\frac{3\pi}{4}$ to $\frac{5\pi}{4}]$ sweeps out the left-hand leaf; $[\frac{5\pi}{4}$ to $\frac{7\pi}{4}]$ sweeps out the upper-hand leaf; $[\frac{7\pi}{4}$ to $\frac{9\pi}{4}]$ sweeps out the right-hand leaf. We can also sweep out the right-hand leaf with $[-\frac{\pi}{4}$ to $\frac{\pi}{4}]$ and this is the one we choose. Hence the **Area** is $\frac{1}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} r^2 d\theta = \frac{1}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos^2(2\theta)\, d\theta = \frac{1}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1+\cos(4\theta)}{2} d\theta =$ $\frac{1}{4}\Big(\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} d\theta + \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos(4\theta)\, d\theta\Big)$. Do the pieces: $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} d\theta = \theta\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{\pi}{4} - (-\frac{\pi}{4}) = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$; $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos(4\theta)\, d\theta = \frac{1}{4}\sin(4\theta)\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{1}{4}\big(\sin\pi - \sin(-\pi)\big) = 0$. The **Area** is $\frac{1}{4}\cdot\big(\frac{\pi}{2}+0\big) = \frac{\pi}{8}$.

\# 7. Area shared by the circles $r = 2\cos\theta$ and $r = 2\sin\theta$.

By drawing the graph, you see each circle can be swept out once by letting θ run from 0 to π. The polar coordinates of the intersection point can be found by solving $2\cos\theta = 2\sin\theta$, or $\tan\theta = 1$ or $\theta = \frac{\pi}{4}$. While there are many solutions to the equation $\tan\theta = 1$, they are all obtained by adding integer multiples of π to $\frac{\pi}{4}$ and we see that the only one between 0 and π is $\frac{\pi}{4}$. Hence the **Area** is $\frac{1}{2}\int_0^{\frac{\pi}{4}} (2\sin\theta)^2 d\theta + \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (2\cos\theta)^2 d\theta =$ $2\int_0^{\frac{\pi}{4}} \sin^2\theta\, d\theta + 2\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2\theta\, d\theta$. Do the pieces: $\int_0^{\frac{\pi}{4}} \sin^2\theta\, d\theta = \int_0^{\frac{\pi}{4}} \frac{1-\cos(2\theta)}{2} d\theta =$ $\frac{1}{2}\Big(\int_0^{\frac{\pi}{4}} d\theta - \int_0^{\frac{\pi}{4}} \cos(2\theta) d\theta\Big) = \frac{1}{2}\Big(\theta\Big|_0^{\frac{\pi}{4}} - \frac{1}{2}\sin(2\theta)\Big|_0^{\frac{\pi}{4}}\Big) = \frac{1}{2}\Big(\frac{\pi}{4} - \frac{1}{2}\big(\sin\frac{\pi}{2} - \sin 0\big)\Big) = \frac{1}{2}\Big(\frac{\pi}{4} - \frac{1}{2}\Big) = \frac{\pi}{8} - \frac{1}{4}$; $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2\theta\, d\theta = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1+\cos(2\theta)}{2} d\theta = \frac{1}{2}\Big(\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\theta + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos(2\theta) d\theta\Big) = \frac{1}{2}\Big(\theta\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} + \frac{1}{2}\sin(2\theta)\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}\Big) = \frac{1}{2}\Big(\big(\frac{\pi}{2} - \frac{\pi}{4}\big) + \frac{1}{2}\big(\sin\pi - \sin\frac{\pi}{2}\big)\Big) = \frac{1}{2}\Big(\frac{\pi}{4} + \frac{1}{2}(0-1)\Big) = \frac{\pi}{8} - \frac{1}{4}$. Hence the **Area** is $2\big(\frac{\pi}{8} - \frac{1}{4}\big) + 2\big(\frac{\pi}{8} - \frac{1}{4}\big) = \frac{\pi}{2} - 1$

\# 11. Inside the lemniscate $r^2 = 6\cos(2\theta)$ and outside the circle $r = \sqrt{3}$. The lemniscate can be graphed as follows. It is actually two equations $r = \pm\sqrt{6\cos(2\theta)}$. Intervals where $\cos(2\theta) < 0$ are excluded: these intervals are $(\frac{\pi}{4}, \frac{3\pi}{4})$, $(\frac{5\pi}{4}, \frac{7\pi}{4})$, etc. The right-hand loop of the lemniscate is traced out by starting θ at $-\frac{\pi}{4}$ and going to $\frac{\pi}{4}$. The entire lemniscate can be described as the graph of $r = \sqrt{6\cos(2\theta)}$ where θ runs over the intervals $[-\frac{\pi}{4}, \frac{\pi}{4}]$ and $[\frac{3\pi}{4}, \frac{5\pi}{4}]$. Next we need to find the four points of intersection, so solve $6\cos(2\theta) = r^2 = 3$ or $\cos(2\theta) = \frac{1}{2}$ so $2\theta = \frac{\pi}{3} + 2k\pi$, k an integer, or $2\theta = -\frac{\pi}{3} + 2k\pi$. Hence $\theta = \pm\frac{\pi}{6} + k\pi$ and the four points are $\theta = \frac{\pi}{6}$ (1st quadrant); $-\frac{\pi}{6}$ (4th quadrant); $\frac{7\pi}{6}$ (3rd quadrant); and $\frac{5\pi}{6}$ (2nd quadrant).

The desired **Area** is $\frac{1}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \big(6\cos(2\theta) - (\sqrt{3})^2\big)d\theta + \frac{1}{2}\int_{\frac{5\pi}{6}}^{\frac{7\pi}{6}} \big(6\cos(2\theta) - (\sqrt{3})^2\big)d\theta =$ $3\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \cos(2\theta)\, d\theta - \frac{3}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} d\theta + 3\int_{\frac{5\pi}{6}}^{\frac{7\pi}{6}} \cos(2\theta)\, d\theta - \frac{3}{2}\int_{\frac{5\pi}{6}}^{\frac{7\pi}{6}} d\theta = 3\sin(2\theta)\Big|_{-\frac{\pi}{6}}^{\frac{\pi}{6}} - \frac{3}{2}\theta\Big|_{-\frac{\pi}{6}}^{\frac{\pi}{6}} +$ $3\sin(2\theta)\Big|_{\frac{5\pi}{6}}^{\frac{7\pi}{6}} - \frac{3}{2}\theta\Big|_{\frac{5\pi}{6}}^{\frac{7\pi}{6}} = 3\big(\sin(\frac{\pi}{3}) - \sin(-\frac{\pi}{3})\big) - \frac{3}{2}(\frac{\pi}{6} - (-\frac{\pi}{6})) + 3\big(\sin(\frac{7\pi}{3}) - \sin(\frac{5\pi}{3})\big) - \frac{3}{2}(\frac{7\pi}{6} - (\frac{5\pi}{6}))$. Now $\sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$; $\sin(-\frac{\pi}{3}) = -\sin(\frac{\pi}{3})$. Since $\frac{7\pi}{3} = 2\pi + \frac{\pi}{3}$, $\sin(\frac{7\pi}{3}) = \sin(\frac{\pi}{3})$ and similarly, $\sin(\frac{5\pi}{3}) = \sin(-\frac{\pi}{3}) = -\sin(\frac{\pi}{3})$. Hence **Area**$= 6\frac{\sqrt{3}}{2} - \frac{3}{2}\frac{2\pi}{3} = 3\sqrt{3} - \pi$.

\# 21. Find the length of the cardioid $r = 1 + \cos\theta$.

The graph is swept out once as θ runs from 0 to 2π. **Length**$= \int_0^{2\pi} \sqrt{r^2 + (r')^2}\, d\theta$. Compute as follows. $\frac{dr}{d\theta} = -\sin\theta$, so $(r')^2 = \sin^2\theta$ so $r^2 + (r')^2 = (1+\cos\theta)^2 + \sin^2\theta =$ $1 + 2\cos\theta + \cos^2\theta + \sin^2\theta = 2 + 2\cos\theta = 2(1+\cos\theta) = 4\frac{1+\cos\theta}{2} = 4\cos^2(\frac{\theta}{2})$. Hence **Length** $= \int_0^{2\pi} \sqrt{4\cos^2(\frac{\theta}{2})}\, d\theta = 2\int_0^{2\pi} \Big|\cos(\frac{\theta}{2})\Big|\, d\theta = 2\int_0^{\pi} \cos(\frac{\theta}{2})\, d\theta - 2\int_{\pi}^{2\pi} \cos(\frac{\theta}{2})\, d\theta =$ $2\Big(2\sin\Big(\frac{\theta}{2}\Big)\Big|_0^{\pi}\Big) - 2\Big(2\sin\Big(\frac{\theta}{2}\Big)\Big|_{\pi}^{2\pi}\Big) = 2\Big(2\sin\Big(\frac{\pi}{2}\Big) - 2\sin(0)\Big) - 2\Big(2\sin(\pi) - 2\sin\Big(\frac{\pi}{2}\Big)\Big) = 8.$

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25. Find the length of the curve $r = \cos^3\left(\frac{\theta}{3}\right)$ $0 \le \theta \le \frac{\pi}{4}$. There is no need to graph this curve since we are told the limits of integration, but just for the record, here is the graph.

Next compute $\frac{dr}{d\theta} = 3\cos^2\left(\frac{\theta}{3}\right)\left(-\sin\left(\frac{\theta}{3}\right)\frac{1}{3}\right) = -\sin\left(\frac{\theta}{3}\right)\cos^2\left(\frac{\theta}{3}\right)$. Hence $r^2 + (r')^2 = \cos^6\left(\frac{\theta}{3}\right) + \sin^2\left(\frac{\theta}{3}\right)\cos^4\left(\frac{\theta}{3}\right) = \cos^4\left(\frac{\theta}{3}\right)\left(\cos^2\left(\frac{\theta}{3}\right) + \sin^2\left(\frac{\theta}{3}\right)\right) = \cos^4\left(\frac{\theta}{3}\right)$. Between 0 and $\frac{\pi}{4}$, $\cos\left(\frac{\theta}{3}\right) > 0$, so **Length** $= \int_0^{\frac{\pi}{4}} \sqrt{r^2 + (r')^2}\, d\theta = \int_0^{\frac{\pi}{4}} \cos^2\left(\frac{\theta}{3}\right) d\theta = \int_0^{\frac{\pi}{4}} \frac{1 + \cos(\frac{2\theta}{3})}{2} d\theta = \frac{1}{2}\int_0^{\frac{\pi}{4}} d\theta + \frac{1}{2}\int_0^{\frac{\pi}{4}} \cos\left(\frac{2\theta}{3}\right) d\theta = \frac{1}{2}\theta\Big|_0^{\frac{\pi}{4}} + \frac{1}{2}\left(\frac{3}{2}\sin\left(\frac{2\theta}{3}\right)\Big|_0^{\frac{\pi}{4}}\right) = \frac{1}{2}\left(\frac{\pi}{4} - 0\right) + \frac{3}{4}\left(\sin\left(\frac{\pi}{6}\right) - \sin(0)\right) = \frac{\pi}{8} + \frac{3}{8}$.

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29. Find the surface area generated by revolving $r = \sqrt{\cos(2\theta)}$, $0 \le \theta \le \frac{\pi}{4}$ about the y–axis. Again the graph is not necessary but is included so you may practice if you wish.

Compute as follows. $\frac{dr}{d\theta} = \frac{-(\sin(2\theta))2}{2\sqrt{\cos(2\theta)}} = -\frac{\sin(2\theta)}{\sqrt{\cos(2\theta)}}$; $r^2 + (r')^2 = \cos(2\theta) + \frac{\sin^2(2\theta)}{\cos(2\theta)} = \frac{\cos^2(2\theta) + \sin^2(2\theta)}{\cos(2\theta)} = \frac{1}{\cos(2\theta)}$. Hence $\sqrt{r^2 + (r')^2} = \frac{1}{\sqrt{\cos(2\theta)}}$. From the book we have the formula, **SurfaceArea** $= 2\pi\int_0^{\frac{\pi}{4}} r\cos\theta\sqrt{r^2 + (r')^2}\, d\theta =$

$$2\pi\int_0^{\frac{\pi}{4}} \sqrt{\cos(2\theta)}\cos\theta\frac{1}{\sqrt{\cos(2\theta)}}\, d\theta = 2\pi\int_0^{\frac{\pi}{4}} \cos\theta\, d\theta = 2\pi\sin\theta\Big|_0^{\frac{\pi}{4}} = 2\pi\left(\sin\left(\frac{\pi}{4}\right) - \right.$$

$\sin(0)\Big) = 2\pi\frac{\sqrt{2}}{2} = \pi\sqrt{2}$

\# 31. Find the surface area generated by revolving $r^2 = \cos(2\theta)$ about the x–axis. This time a graph is helpful in determining the limits of integration.

The function $\cos(2\theta)$ vanishes at $-\frac{\pi}{4}$, $\frac{\pi}{4}$, $\frac{3\pi}{4}$, $\frac{5\pi}{4}$, etc. so the curve can be described as $r = \sqrt{\cos(2\theta)}$ for $[-\frac{\pi}{4}, \frac{\pi}{4}]$ and $[\frac{3\pi}{4}, \frac{5\pi}{4}]$.

A further problem occurs. To get the surface of revolution, we should only rotate the top half or the bottom half of the curve. The right-hand branch can be swept out by letting θ run from 0 to $\frac{\pi}{4}$, which sweeps out the top half of the right-hand piece. The top half of the left-hand piece needs θ to run from $\frac{3\pi}{4}$ to π.

The next step is to compute $\sqrt{r^2 + (r')^2}$. We just did this in #29 and we got $\sqrt{r^2 + (r')^2} = \dfrac{1}{\sqrt{\cos(2\theta)}}$.

Hence **SurfaceArea** $= 2\pi\displaystyle\int_0^{\frac{\pi}{4}} \sqrt{\cos(2\theta)}\ \sin\theta\frac{d\theta}{\sqrt{\cos(2\theta)}} + 2\pi\int_{\frac{3\pi}{4}}^{\pi} \sqrt{\cos(2\theta)}\ \sin\theta\frac{d\theta}{\sqrt{\cos(2\theta)}} =$ $2\pi\displaystyle\int_0^{\frac{\pi}{4}} \sin\theta\, d\theta + 2\pi\int_{\frac{3\pi}{4}}^{\pi} \sin\theta\, d\theta = 2\pi\left((-\cos\theta)\Big|_0^{\frac{\pi}{4}} + (-\cos\theta)\Big|_{\frac{3\pi}{4}}^{\pi}\right) = 2\pi\Bigg(\Big((-\cos(\frac{\pi}{4})) -$ $(-\cos(0))\Big) + \Big((-\cos(\pi)) - (-\cos(\frac{3\pi}{4}))\Big)\Bigg)$. Since $\cos(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$; $\cos(0) = 1$; $\cos(\pi) = -1$; and $\cos(\frac{3\pi}{4}) = -\frac{\sqrt{2}}{2}$ we get **SurfaceArea** $= 2\pi(2 - \sqrt{2})$.

\# 37. Find the centroid of the region enclosed by the cardioid $r = a(1+\cos\theta)$. Look at #21 for the graph of the cardioid with $a = 1$ and recall that the cure is swept out once as θ runs from 0 to 2π.

To find the centroid we first calculate the area: **Area** $= \frac{1}{2}\displaystyle\int_0^{2\pi} \big(a(1+\cos\theta)\big)^2 d\theta =$ $\frac{a^2}{2}\displaystyle\int_0^{2\pi}(1+2\cos\theta+\cos^2\theta)d\theta = \frac{a^2}{2}\Big(\int_0^{2\pi} d\theta + 2\int_0^{2\pi}\cos\theta d\theta + \int_0^{2\pi}\cos^2\theta d\theta\Big) = \frac{a^2}{2}\Big(\int_0^{2\pi} d\theta +$ $2\displaystyle\int_0^{2\pi}\cos\theta\, d\theta + \int_0^{2\pi}\frac{1+\cos(2\theta)}{2}\, d\theta\Big) = \frac{a^2}{2}\Big(\frac{3}{2}\int_0^{2\pi} d\theta + 2\int_0^{2\pi}\cos\theta\, d\theta + \frac{1}{2}\int_0^{2\pi}\cos(2\theta)\, d\theta\Big)$. The integral involving $\cos\theta$ is 0 as is the integral involving $\cos(2\theta)$ so **Area** $= \dfrac{6\pi a^2}{4}$. Now we turn to the moments.

The **moment about the x–axis** is $\frac{1}{3}\int_0^{2\pi} r^3 \sin\theta\, d\theta = \frac{1}{3}\int_0^{2\pi} a^3(1+\cos\theta)^3 \sin\theta\, d\theta$. Substitute $u = 1+\cos\theta$, $du = -\sin\theta\, d\theta$. $\int_0^{2\pi} a^3(1+\cos\theta)^3\sin\theta\, d\theta = -\int_2^2 a^3u^3\, du = 0$. Once can also argue from symmetry: there is as much of the curve above the x–axis as below it.

The **moment about the y–axis** is $\frac{1}{3}\int_0^{2\pi} r^3\cos\theta\, d\theta = \frac{1}{3}\int_0^{2\pi} a^3(1+\cos\theta)^3\cos\theta\, d\theta = \frac{a^3}{3}\int_0^{2\pi}(1+3\cos\theta+3\cos^2\theta+\cos^3\theta)\cos\theta d\theta = \frac{a^3}{3}\int_0^{2\pi}(\cos\theta+3\cos^2\theta+3\cos^3\theta+\cos^4\theta)d\theta =$.

We can do $\int \cos^4\, d\theta$ by parts as follows. Let $u = \cos^3\theta$, $dv = \cos\theta\, d\theta$. Then $du = -3\cos^2\theta\sin\theta d\theta$ and $v = \sin\theta$. Hence $\int \cos^4\, d\theta = \cos^3\theta\sin\theta - \int(\sin\theta(-3\cos^2\theta\sin\theta)d\theta = \cos^3\theta\sin\theta + 3\int\cos^2\theta\sin^2\theta\, d\theta$. Now write $\sin^2\theta = 1-\cos^2\theta$ so $\int\cos^2\theta\sin^2\theta\, d\theta = \int\cos^2\theta d\theta - \int\cos^4\theta d\theta$. Plug back in and solve for $\int\cos^4\theta d\theta$: $4\int\cos^4\theta d\theta = \cos^3\theta\sin\theta + 3\int\cos^2\theta\, d\theta$, or $\int\cos^4\theta\, d\theta = \frac{\cos^3\theta\sin\theta}{4} + \frac{3}{4}\int\cos^2\theta\, d\theta$.

A similar Integration by Parts and solving for the integral gives $\int\cos^3\theta d\theta = \frac{\cos^2\theta\sin\theta}{3} + \frac{2}{3}\int\cos\theta\, d\theta$.

Hence $\int_0^{2\pi}(\cos\theta + 3\cos^2\theta + 3\cos^3\theta + \cos^4\theta)\, d\theta = \int_0^{2\pi}(3\cos\theta + \frac{15}{4}\cos^2\theta)\, d\theta + \cos^2\theta\sin\theta\Big|_0^{2\pi} + \frac{\cos^3\theta\sin\theta}{4}\Big|_0^{2\pi}$. Further $\int_0^{2\pi}\cos\theta d\theta = 0$ and $\int_0^{2\pi}\cos^2\theta d\theta = \int_0^{2\pi}\frac{1+\cos(2\theta)}{2}d\theta = \pi$. Hence the **moment about the y–axis** $= \frac{a^3}{3}\frac{15\pi}{4} = \frac{5\pi a^3}{4}$.

Therefore, the x coordinate of the center of mass is $\frac{\frac{5\pi a^3}{4}}{\frac{6\pi a^2}{4}} = \frac{5a}{6}$. The y coordinate of the center of mass is 0.