

37)

$$\int \frac{dx}{2 + (x - 1)^2} = ?$$

Let $u = x - 1$ then $du = dx$ hence

$$\int \frac{dx}{2 + (x - 1)^2} = \int \frac{du}{2 + u^2} = \frac{1}{\sqrt{2}} \tan^{-1} \frac{u}{\sqrt{2}} = \frac{1}{\sqrt{2}} \tan^{-1} \frac{(x - 1)}{\sqrt{2}}$$

39)

$$\int \frac{dx}{(2x - 1)\sqrt{(2x - 1)^2 - 4}} = ?$$

Let $u = 2x - 1$ then $du = 2dx$ hence $\int \frac{dx}{(2x - 1)\sqrt{(2x - 1)^2 - 4}} = \frac{1}{2} \int \frac{du}{u\sqrt{u^2 - 4}} = \frac{1}{2} \frac{1}{2} \sec^{-1} \left| \frac{u}{2} \right|$

41)

$$\int_{-\pi/2}^{\pi/2} \frac{2 \cos \theta d\theta}{1 + (\sin \theta)^2} = ?$$

Let $u = \sin \theta$ then $du = \cos \theta d\theta$ hence $\int_{-\pi/2}^{\pi/2} \frac{2 \cos \theta d\theta}{1 + (\sin \theta)^2} = \int \frac{2du}{1 + u^2} = 2 \tan^{-1} u$

43)

$$\int_0^{\ln \sqrt{3}} \frac{e^x dx}{1 + e^{2x}} = ?$$

Let $u = e^x$ then $du = e^x dx$ hence $\int_0^{\ln \sqrt{3}} \frac{e^x dx}{1 + e^{2x}} = \int \frac{du}{1 + u^2} = \tan^{-1} u = \tan^{-1} e^x \Big|_0^{\ln \sqrt{3}}$

45)

$$\int \frac{y dy}{\sqrt{1 - y^4}} = ?$$

Let $u = y^2$ then $du = 2y dy$ hence $\int \frac{y dy}{\sqrt{1 - y^4}} = \frac{1}{2} \int \frac{du}{\sqrt{1 - u^2}} = \frac{1}{2} \sin^{-1} u = \frac{1}{2} \sin^{-1} y^2$.