[11pt]article document
p. 465
5) $y = \ln 3x, y^{'} = 1/x$
#11) $y = \ln(\theta + 1), y^{'} = 1/(\theta + 1)$
#15) $y = t(\ln t)^2, y^{'} = (\ln t)^2 + 2\ln t$
#19) $y = \ln t/t, y^{'} = 1/t^2 - \ln t/t^2$
21) $y = \ln x/(1 + \ln x), y^{'} = 1/x(1 + \ln x) - \ln x/x(1 + \ln x)^2$
25) $y = \theta(\sin(\ln\theta)) + \cos(\ln\theta)),$

$$y^{'} = \sin(\ln\theta) + \cos(\ln\theta)) + \cos(\ln\theta) - \sin(\ln\theta)$$

37) $y = \sqrt{x(x+1)}$

$$\ln y = \frac{1}{2}\{\ln x + \ln(x+1)\}$$

differentiation yields:

$$y^{'}/y = \frac{1}{2}\{1/x + 1/(x+1)\}$$

thus

$$y^{'} = \frac{1}{2}y\{1/x + 1/(x+1)\} = \frac{1}{2}\sqrt{x(x+1)}\{1/x + 1/(x+1)\}.$$

49)

$$y = (\frac{x(x-2)}{x^2+1})^{1/3}$$

hence

$$\ln y = \frac{1}{3}\{\ln x + \ln(x-2) - \ln(x^2+1)\}$$

differentiation yields:

$$y^{'}/y = \frac{1}{3}\{1/x + 1/(x-2) - 1/(x^2+1)\}$$

so

$$y^{'} = \frac{1}{3}(\frac{x(x-2)}{x^2+1})^{1/3}\{1/x + 1/(x-2) - 1/(x^2+1)\}$$

53)

$$\int \frac{2y}{y^2-25}dy = ?$$

Let $u = y^2 - 25$ then $du = 2ydy$

$$\int \frac{2y}{y^2-25}dy = \int \frac{du}{u} = \ln|u| + C = \ln|y^2-25| + C$$

53)

$$\int_1^2 \frac{2\ln x}{x}dx = ?$$

Set $u = \ln x$ then $du = dx/x$

$$\int_1^2 \frac{2\ln x}{x}dx = 2\int udu = u^2 = (\ln x)^2|_1^2 = (\ln 2)^2 - (\ln 1)^2 = (\ln 2)^2$$

59)

$$\int_2^4 \frac{dx}{x(\ln x)^2} = ?$$

Let $u = \ln x$ then $du = dx/x$ eqnarray* $\int_2^4 \frac{dx}{x(\ln x)^2} = \int u^{-2} du = -u^{-1} = -(\ln x)^{-1}|_2^4 = \frac{1}{\ln 4} - \frac{1}{\ln 2}$
61)

$$\int \frac{3\sec^2 t}{6 + 3\tan t} dt = ?$$

Let $u = 6 + 3tant$ then $du = 3sec^2 t$ and

$$\int \frac{3\sec^2 t}{6 + 3tant} dt = \int \frac{du}{u} = \ln|u| + C = \ln|6 + 3\tan t| + C$$

p. 472
1a) $e^{\ln 7.2} == 7.2$; 1b) $e^{-\ln x^2} = 1/x^2$; 1c) $e^{\ln x - \ln y} = e^{\ln x}/e^{\ln y} = x/y$.
#3a)

$$2\ln\sqrt{e} = 2\ln e^{1/2} = 2.\frac{1}{2}\ln e = 1.$$

3b) $\ln(\ln e^e) = \ln(e\ln e) = \ln e = 1$.
3c) $\ln(e^{-x^2-y^2}) = -x^2 - y^2$.
5) $\ln y = 2t + 4, y = ?$ Take exponential on both sides, we get:

$$y = e^{2t+4}.$$

9) $\ln(y-1) - \ln 2 = x + \ln x, y = ?$ Take exponential on both sides:

$$e^{\ln(y-1)-\ln 2} = e^{x+\ln x}$$

which is equivalent to

$$\frac{y-1}{2} = xe^x$$

hence

$$y = 2xe^x + 1.$$

13a) $e^{-0.3t} = 27, t = ?$ Take ln on both sides:

$$-0.3t = \ln 27, t = -(3\ln 3)/0.3 = -10\ln 3.$$

13b) $e^{kt} = 1/2$. Take ln:

$$kt = -\ln 2, t = -\ln 2/k.$$

13c) $e^{(\ln 0.2)t} = 0.4$. Simplify:

$$(0.2)^t = 0.4$$

then take ln:

$$(\ln 0.2)t = \ln 0.4$$

so $t = \ln 0.2/\ln 0.4$.
19) $y = e^{5-7x}$ then $y^{'} = -7e^{5-7x}$.
23) $y = (x^2 - 2x + 2)e^x$ then $y^{'} = (2x-2)e^x + (x^2 - 2x + 2)e^x$.
27) $y = \cos(e^{-\theta^2})$ then $y^{'} = -\sin(e^{-\theta^2})e^{-\theta^2}(-2\theta) = 2\theta e^{-\theta^2}\cos(e^{-\theta^2})$.
37) $\ln y = e^y \sin x$ then

$$y^{'}/y = e^y y^{'} \sin x + e^y \cos x$$

hence

$$y' = ye^y y' \sin x + ye^y \cos x$$

which is equivalent to

$$y'(1 - ye^y \sin x) = ye^y \cos x.$$

Solving for y':

$$y' = \frac{ye^y \cos x}{1 - ye^y \sin x}.$$

41) $\int (e^{3x} + 5e^{-x})dx = e^{3x}/3 - 5e^{-x}$.
49)

$$\int \frac{e^{\sqrt{r}}}{\sqrt{r}} dr = ?$$

Let $u = \sqrt{r}$ then $du = dr/2\sqrt{r}$ so

$$\int \frac{e^{\sqrt{r}}}{\sqrt{r}} dr = 2 \int e^u du = 2e^u + C = 2e^{\sqrt{r}} + C.$$

55)

$$\int_0^{\pi/4} (1 + e^{\tan\theta}) \sec^2\theta d\theta = ?$$

Let $u = \tan\theta$ then $d\theta = \sec^2\theta$ hence: eqnarray* $\int_0^{\pi/4} (1 + e^{\tan\theta}) \sec^2\theta d\theta = \int (1 + e^u) du$
63) $y' = e^t \sin(e^t - 2), y(\ln 2) = 0$ Then

$$y = \int e^t \sin(e^t - 2) dt$$

set $u = e^t - 2$ then $du = e^t dt$ and

$$y = \int e^t \sin(e^t - 2) dt = \int \sin u du = -\cos u + C = -\cos(e^t - 2) + C.$$

To find C we use the initial condition:

$$0 = y(\ln 2) = -\cos(e^{\ln 2} - 2) + C = -\cos 0 + C = -1 + C$$

hence $C = 1$ and $y = -\cos(e^t - 2) + 1$.
65) $y'' = 2e^{-x}, y(0) = 1, y'(0) = 0$ First integration yields:

$$y' = \int 2e^{-x} dx = -2e^{-x} + C_0$$

Since $y'(0) = 0$, we get:

$$0 = y'(0) = -2e^{-0} + C_0 = -2 + C_0$$

hence $C_0 = 2$. Thus

$$y' = 2 - 2e^{-x}$$

and integration yields:

$$y = \int (2 - 2e^{-x}) dx = 2x + 2e^{-x} + C_1.$$

Using the condition $y(0) = 1$ we get

$$1 = y(0) = 2 + C_1$$

hence $C_1 = -1$ and we have:

$$y = 2x + 2e^{-x} - 1.$$

69) Find the absolute maximum valuse of $f(x) = x^2 \ln(1/x) = -x^2 \ln x$.
Differentiation yields:

$$f^{'}(x) = -2x \ln x - x^2/x = -2x \ln x - x$$

Set $f^{'}(x) = -2x \ln x - x = 0$ we get

$$x(-2 \ln x - 1) = 0$$

and so $x = 0, x = e^{-1/2}$ are the critical points. Absolute maximum occurs at $x = e^{-1/2}$ the absolute maximum value is $-(e^{-1/2})^2 \ln(e^{-1/2}) = -e^{-1}(-1/2) = 1/2e$.

p.480
#13) $y = 5^{\sqrt{x}}$,

$$\ln y = \sqrt{x} \ln 5, \quad y^{'}/y = \frac{\ln 5}{2\sqrt{x}}$$

hence

$$y^{'} = y\frac{\ln 5}{2\sqrt{x}} = 5^{\sqrt{x}} \frac{\ln 5}{2\sqrt{x}}.$$

#17) $y = (\cos \theta)^{\sqrt{2}}$,

$$\ln y = \sqrt{2} \ln(\cos \theta), \quad y^{'}/y = \sqrt{2}\frac{1}{\cos \theta}(-\sin \theta) = -\sqrt{2} \tan \theta$$

so

$$y^{'} = -\sqrt{2} \tan \theta (\cos \theta)^{\sqrt{2}}.$$

#33) $y = \log_5 e^x = \ln e^x / \ln 5 = x / \ln 5$, hence

$$y^{'} = \frac{1}{\ln 5}.$$

#39) $y = (x+1)^x, \quad \ln y = x \ln(x+1)$, hence

$$y^{'}/y = \ln(x+1) + x/(x+1)$$

so

$$y = (x+1)^x \{\frac{x}{x+1} + \ln(x+1)\}.$$

#43) $y = (\sin x)^x, \quad \ln y = x \ln \sin x$, hence

$$y^{'}/y = \ln \sin x + x\frac{1}{\sin x} \cos x = \ln \sin x + x \tan x$$

and so

$$y = (\sin x)^x \{\ln \sin x + x \tan x\}.$$

#47)

$$\int 5^x dx = \frac{5^x}{\ln 5} + C$$

#51)

$$\int_1^{\sqrt{2}} x 2^{x^2} dx =?$$

Let $u = x^2$ then $du = 2xdx$ eqnarray* $\int_1^{\sqrt{2}} x2^{x^2} dx = \frac{1}{2}\int 2^u du = \frac{1}{2}\frac{2^u}{\ln 2}$
#57)

$$\int 3x^{\sqrt{3}} dx = \frac{3}{11+\sqrt{3}} x^{1+\sqrt{3}}.$$

#65) eqnarray* $\int_0^2 \frac{\log_2(x+2)}{x+2} = \int_0^2 \frac{\ln(x+2)}{(x+2)\ln 2}$

p.489
#3)

$$\frac{dy}{dt} = -0.6y, y(0) = 100$$

Then

$$\frac{dy}{y} = -0.6dt$$

integration yields

$$\ln y = -0.6t + C$$

and $\ln 100 = -0.6(0) + C = C$. Thus $\ln y = -0.6t + \ln 100$. Taking exponential yields:

$$y = 100e^{-0.6t}$$

For $t = 1$ we get

$$y = 100e^{-0.6}.$$

#5)

$$\frac{dL}{dx} = -kL$$

Intensity is halved at the depth of 18 feet. Solving the equation yields:

$$L = L_0 e^{-kx}$$

and

$$\frac{1}{2}L_0 = L_0 e^{-k18}$$

so

$$e^{-k18} = 1/2$$

and taking ln yields:

$$-18k = -\ln 2, \quad k = \ln 2/18$$

Thus

$$L = L_0 e^{-(\ln 2)x/18}.$$

If $L(x) = L_0/10$ then

$$1/10 = e^{-(\ln 2)x/18}$$

thus

$$\frac{\ln 2}{18} x = \ln 10$$

and so $x = 18 \ln 10/\ln 2$.
#7) 2^{48}
#13) $A(t) = A_0 e^{.04t}$ thus

$$A(5) = A_0 e^{.2}.$$

If $A(t) = 2A_0$ then

$$2 = e^{.04t}, \ \ln 2 = .04t$$

so $t = \ln 2/.04$. If $A(t) = 3A_0$ then

$$3 = e^{.04t},\ \ln 3 = .04t$$

so $t = \ln 3/.04$.

p. 496

#7)

$$\lim_{t\to 0} \frac{\sin t^2}{t} = \lim_{t\to 0} \frac{2t\cos t^2}{1} = 0.$$

#13)

$$\lim_{\theta\to\pi/2} \frac{1-\sin\theta}{1+\cos 2\theta} = \lim_{\theta\to\pi/2} \frac{-\cos\theta}{-2\sin 2\theta} = \lim_{\theta\to\pi/2} \frac{-\sin\theta}{-4\cos 2\theta} = 0.$$

#15)

$$\lim_{x\to 0} \frac{x^2}{\ln(\sec x)} = \lim_{x\to 0} \frac{2x\sec x}{\tan x \sec x} = \lim_{x\to 0} \frac{2x}{\tan x} = \lim_{x\to 0} \frac{2}{\sec^2 x} = 2.$$

#19)

$$\lim_{x\to(\pi/2)^-} (x-\frac{\pi}{2})\sec x = \lim_{x\to(\pi/2)^-} \frac{(x-\frac{\pi}{2})}{\cos x} = \lim_{x\to(\pi/2)^-} \frac{1}{-\sin x} = -1.$$

#27)

$$\lim_{x\to 0^+} \frac{\ln(x^2+2x)}{\ln x} = \lim_{x\to 0^+} \frac{2(x+1)}{x^2+2x}x = \lim_{x\to 0^+} \frac{2(x+1)}{x+2} = 1.$$

#31)

$$\lim_{x\to\infty} (\ln 2x - \ln(x+1)) = \lim_{x\to\infty} \ln\frac{2x}{x+1} = \ln \lim_{x\to\infty} \frac{2x}{x+1} = \ln \lim_{x\to\infty} \ln\frac{2}{1} = \ln 2.$$

#37)

$$\lim_{x\to\infty} \int_x^{2x} \frac{1}{t}dt = \lim_{x\to\infty} \ln t|_x^{2x} = \lim_{x\to\infty} \ln\frac{2x}{x} = \ln 2.$$

#42)

$$\lim_{x\to\infty} x^2 e^{-x} = \lim_{x\to\infty} \frac{x^2}{e^x} = \lim_{x\to\infty} \frac{2x}{e^x} = \lim_{x\to\infty} \frac{2}{e^x} = 0.$$

#51)

$$\lim_{x\to 0^+} x^x = \lim_{x\to 0^+} e^{x\ln x}.$$

Now

$$\lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \frac{\ln x}{1/x} = \lim_{x\to 0^+} -\frac{1/x}{1/x^2} = \lim_{x\to 0^+} -x = 0.$$

Thus

$$\lim_{x\to 0^+} x^x = \lim_{x\to 0^+} e^{x\ln x} = e^0 = 1.$$

#53) eqnarray* $\lim_{x\to\infty} \sqrt{\frac{9x+1}{x+1}} = \lim_{x\to\infty} \sqrt{\frac{(9x+1)/x}{(x+1)/x}} = \lim_{x\to\infty} \sqrt{\frac{(9+1/x)}{(1+1/x)}}$

#57) (a) is wrong and (b) is correct becuse

$$\lim_{x\to 3} \frac{x-3}{x^2-3}$$

is not of indeterminate form.

#63)

$$\lim_{k\to\infty}(1+\frac{r}{k})^k = \lim_{k\to\infty} e^{k\ln(1+r/k)}$$

and since eqnarray* $\lim_{k\to\infty} k\ln(1+\frac{r}{k}) = \lim_{k\to\infty} \ln(1+\frac{r}{k})/(1/k) = \lim_{k\to\infty} \frac{-r/k^2}{1+r/k}(-k^2)$

p.503

#1) (Look at examples 1, 2, 3, 4 and 5) Slower : a), b), c), e), h); Same rate: g); Faster d).

#3) c) Same rate:

$$\lim_{x\to\infty}\frac{\sqrt{x^4+x^3}}{x^2} = \lim_{x\to\infty}\sqrt{\frac{x^4}{x^4}+\frac{x^3}{x^4}} = \lim_{x\to\infty}\sqrt{1+\frac{1}{x}} = 1.$$

g) Slower:

$$\lim_{x\to\infty}\frac{x^3e^{-x}}{x^2} = \lim_{x\to\infty}\frac{x}{e^x} = 0.$$

23)

$$\lim_{x\to\infty}\frac{n\log_2 n}{n(\log_2 n)^2} = \lim_{x\to\infty}\frac{1}{\log_2 n} = 0;$$

$$\lim_{x\to\infty}\frac{n(\log_2 n)^2}{n^{3/2}} = \lim_{x\to\infty}\frac{(\log_2 n)^2}{n^{1/2}} = 0.$$

23)

$$\int\frac{dx}{\sqrt{9-x^2}} = \sin^{-1}\frac{x}{3}.$$

25)

$$\int\frac{dx}{17+x^2} = \frac{1}{\sqrt{17}}\tan^{-1}\frac{x}{\sqrt{17}}.$$

27)

$$\int\frac{dx}{x\sqrt{5x^2-4}} =?$$

Let $u=\sqrt{5}x$ then $du=\sqrt{5}dx$ then

$$\int\frac{dx}{x\sqrt{5x^2-4}} = \int\frac{du}{u\sqrt{u^2-4}} = \frac{1}{2}\sec^{-1}|\frac{\sqrt{5}x}{2}|$$

29)

$$\int_0^1\frac{4ds}{\sqrt{4-s^2}} = 4\sin^{-1}\frac{x}{2}|_0^1 = 4\sin^{-1}\frac{1}{2} - 4\sin^{-1}0 = 4\pi/6 = 2\pi/3.$$

31)

$$\int_0^2\frac{dt}{8+2t^2} =?$$

Let $u=\sqrt{2}t$ then $du=\sqrt{2}dt$ hence eqnarray* $\int_0^2\frac{dt}{8+2t^2} = \frac{1}{\sqrt{2}}\int\frac{du}{8+u^2} = \frac{1}{\sqrt{2}}\frac{1}{2\sqrt{2}}\tan^{-1}\frac{u}{2\sqrt{2}} = \frac{1}{4}\tan^{-1}\frac{t}{2}|_0^2$

33)

$$\int_{-1}^{-\sqrt{2}/2} \frac{dy}{y\sqrt{4y^2-1}} = ?$$

Let $u = 2y$ then $du = 2dy$ hence eqnarray* $\int_{-1}^{-\sqrt{2}/2} \frac{dy}{y\sqrt{4y^2-1}} = \int \frac{du}{u\sqrt{u^2-1}} = \sec^{-1} u$

35)

$$\int \frac{3dr}{\sqrt{1-4(r-1)^2}} = ?$$

Let $u = 2(r-1)$ then $du = 2dr$ and

$$\int \frac{3dr}{\sqrt{1-4(r-1)^2}} = \frac{3}{2}\int \frac{du}{\sqrt{1-u^2}} = \frac{3}{2}\sin^{-1}|u| = \frac{3}{2}\sin^{-1} 2(r-1).$$