Math 126
Fall 1997
Final Examination
Dec. 16, 1997

- This test runs for two hours.
- All work is to be your own.
- Calculators are not to be used.

Name: _____ **Professor:** Taylor

If Taylor is not the name of your professor, please raise your hand.

Grading on this examination.

READ CAREFULLY BEFORE BEGINNING.

The first three questions will require you to demonstrate knowledge of a correct procedure for arriving at the answer. Please circle the answer you want us to consider. The correct answer is not enough for full credit, you must indicate how you arrived at it. Please mark out any work on a problem that you do not want us to consider. Questions 4 through 24 are multiple choice and are either correct or wrong. Please put an **X** through your answer to each multiple choice question in the space provided on this page as in the **Sample** below.

Sample (a) (b) (X) (d) (e)

4. (a) (b) (c) (d) (e)
5. (a) (b) (c) (d) (e)
6. (a) (b) (c) (d) (e)
7. (a) (b) (c) (d) (e)
8. (a) (b) (c) (d) (e)
9. (a) (b) (c)
10. (a) (b) (c)
11. (a) (b) (c)
12. (a) (b) (c) (d) (e)
13. (a) (b) (c) (d) (e)
14. (a) (b) (c) (d) (e)
15. (a) (b) (c) (d) (e)
16. (a) (b) (c) (d) (e)
17. (a) (b) (c) (d) (e)
18. (a) (b) (c) (d) (e)
19. (a) (b) (c) (d) (e)
20. (a) (b) (c) (d) (e)
21. (a) (b) (c) (d) (e)
22. (a) (b) (c) (d) (e)
23. (a) (b) (c) (d) (e)
24. (a) (b) (c) (d) (e)

Used for grading.

1. _____
2. _____
3. _____

M.C. _____

Total: _____

1. (15pt) Find the area inside the 3–leafed rose, $r = \sin(3\theta)$.

2. (15pt) Find the arclength of the curve $r = \sec\theta$ with $0 \leq \theta \leq \frac{\pi}{4}$.

3. (15pt) Find the surface area of the surface obtained by rotating the piece of $r^2 = 1+\cos(2\theta)$ in the first quadrant around the x–axis. (The graph is that of the entire curve.)

4. (5pt) Which function below is the inverse function to $f(x) = e^{2x}$?
(a) $e^{-x} + \ln|x|$ (b) $e^{-x/2}$ (c) $\sqrt{\ln x}$ (d) $\frac{1}{2}\ln x$ (e) $\ln(x/2)$

5. (5pt) Which substitution reduces the integral $\int \frac{dx}{\sqrt{4-x^2}}$ to the integral $\int du$?
(a) $x = 2\sin u$ (b) $x = \frac{1}{2}\tan u$ (c) $x = \sin u + \cos u$ (d) $u = 2\sin x$ (e) $x = \sqrt{2}\cos u$

6. (5pt) Use Integration by Parts to show $\int_0^{\frac{\pi}{2}} \cos^{10} x\, dx$ equals one of the numbers below.
(a) $\frac{\pi}{2}$ (b) $(-10)\int_0^{\frac{\pi}{2}} \cos^9 x \sin x\, dx$ (c) $\frac{9}{10}\int_0^{\frac{\pi}{2}} \cos^8 x\, dx$ (d) 0 (e) $\int_0^{\frac{\pi}{2}} \cos^8 x\, dx$

7. (5pt) The value of $\lim_{t\to\infty} t \cdot \sin\left(\frac{1}{2t}\right) =$
(a) 4 (b) 2 (c) 1 (d) $\frac{1}{4}$ (e) $\frac{1}{2}$

8. (5pt) Which function below is the solution to the initial value problem $y' = 3y$, $y(1) = 1$?
(a) $y(x) = \frac{e^3}{e^{3x}}$ (b) $y(t) = \frac{e^{3t}}{e^3}$ (c) $y(s) = e^{3s} - e^3 + 1$ (d) $y(x) = x^3$ (e) $y(t) = e^{3t}$

9. (5pt) Indicate which one of the statements below is true. The series $\sum_{n=0}^{\infty} \frac{1}{n^2+3}$
(a) absolutely converges (b) conditionally converges (c) diverges

10. (5pt) Indicate which one of the statements below is true. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n^2+n}}$
(a) absolutely converges (b) conditionally converges (c) diverges

11. (5pt) Indicate which one of the statements below is true. The series $\sum_{n=3}^{\infty} \frac{1}{n \ln n}$
(a) absolutely converges (b) conditionally converges (c) diverges

12. (5pt) Indicate which one of the statements below is true. The series

$$\sum_{n=0}^{\infty} \frac{2n+1}{(n^2+1)(n^2+2n+2)}$$

(a) has value 2 (b) has value 1 (c) has value 4 (d) diverges (e) has value 3

13. (5pt) Which series below is the Taylor series for the function $\ln x$ at 2?

(a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{x^n}{n}$ (b) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{(x-2)^{2n}}{(2n)!}$ (c) $\ln 2 + \displaystyle\sum_{n=1}^{\infty}(-1)^{n+1} \frac{(x-2)^n}{n2^n}$

(d) $\displaystyle\sum_{n=2}^{\infty}(-1)^{n-1} \frac{(x-2)^{n-3}}{\ln 2}$ (e) $\displaystyle\sum_{n=0}^{\infty} \frac{(x-2)^n}{n!}$

14. (5pt) The MacLaurin Series for $(1+x^3)^{\frac{1}{3}}$ starts out

(a) $1 + \frac{1}{3}x - \frac{1}{9}x^2 + \frac{5}{81}x^3 \cdots$ (b) $\frac{1}{3}x^3 - \frac{1}{9}x^6 + \frac{5}{81}x^9 \cdots$ (c) $\frac{1}{3}x - \frac{1}{9}x^2 + \frac{5}{81}x^3 \cdots$

(d) $\sqrt[3]{1+x^3} + \cdots$ (e) $1 + \frac{1}{3}x^3 - \frac{1}{9}x^6 + \frac{5}{81}x^9 \cdots$

15. (5pt) The difference $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^2}{2^n} - \sum_{n=1}^{4}(-1)^{n+1}\frac{n^2}{2^n}$ is

(a) positive and less than $\frac{25}{32}$ (b) negative and greater than $-\frac{49}{1024}$
(c) negative and greater than $-\frac{25}{32}$ (d) greater than $-\frac{1}{1024}$ and less than $\frac{1}{1024}$
(e) positive and less than $\frac{49}{1024}$

16. (5pt) The partial fraction decomposition for $\frac{x^5+4x^3-4x^2+2x-3}{x^2(x^2+1)}$ is

(a) $x + \frac{3}{x^2} + \frac{x-1}{x^2+1}$ (b) $x^2 + \frac{2}{x} - \frac{5}{x^2} + \frac{x-1}{x^2+1}$ (c) $x - \frac{3}{x^2} + \frac{x-1}{x^2+1}$
(d) $x + \frac{2}{x} - \frac{3}{x^2} + \frac{x-1}{x^2+1}$ (e) $\frac{2}{x} - \frac{5}{x^2} + \frac{x-1}{x^2+1}$

17. (5pt) $\int_0^1 xe^x dx =$

(a) 1 (b) 0 (c) 3 (d) 4 (e) 2

18. (5pt) The radius of convergence of $\sum_{n=0}^{\infty} \frac{1 \cdot 3 \cdots (2n-1)}{2 \cdot 4 \cdots (2n)} x^n$ is

(a) ∞ (b) 8 (c) 3 (d) 2 (e) 1

19. (5pt) $\frac{d\, 3^x}{dx} =$

(a) $x3^{x-1}$ (b) $\ln(3^x)$ (c) $(\ln 3)3^x$ (d) $\frac{3^x}{\ln 3}$ (e) 3^x

20. (5pt) $\int_0^{\infty} \frac{dx}{x^2+1} =$

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) diverges (d) $\arctan 4$ (e) 0

21. (5pt) $\int_0^{2} \frac{dx}{x-1} =$

(a) 2 (b) $\ln 3$ (c) $\ln 2$ (d) diverges (e) 0

22. (5pt) Which of the functions below grows the most slowly?

(a) $x^3 - 3x$ (b) $\frac{x^2}{4}$ (c) $x^6 + 3x^5 + 4$ (d) $x \ln x$ (e) $\ln\left(e^{x^6}\right)$

23. (5pt) $\frac{d \operatorname{arcsec}(x^2)}{dx} =$

(a) $\frac{2x}{\sqrt{1-x^4}}$ (b) $\frac{1}{\sqrt{1-x^4}}$ (c) $\frac{1}{x^2\sqrt{x^4-1}}$ (d) $\frac{x^2}{\sqrt{1-x^4}}$ (e) $\frac{2}{x\sqrt{x^4-1}}$

24. (5pt) Which function below is a solution to the differential equation $y' - x^2 y = 0$?

(a) $y = 3 + e^{\frac{x^3}{3}}$ (b) $y = \frac{3}{2}e^{\frac{x^3}{2}}$ (c) $y = 2e^{\frac{x^3}{3}}$ (d) $y = 3e^{\frac{x^2}{2}}$ (e) $y = 2 + e^{\frac{x^2}{2}}$

4- d
5- a
6- c
7- e
8- b
9- a
10- b
11- c
12- b
13- c
14- e
15- a
16- d
17- a
18- e
19- c
20- b
21- d
22- d
23- e
24- c