1. The region bounded by the curve $y = \frac{1}{x^2}$, the lines $x = \frac{2}{3}$ and $x = \frac{3}{2}$, and the x-axis, is revolved about the y-axis. The volume of the solid obtained in this way is

(A) $2\pi \ln \left(\frac{5}{6}\right)$

(B) $2\pi \ln 5$

(C) $\frac{5\pi}{3}$

(D) $4\pi (\ln 3 - \ln 2)$

(E) $2\pi e^{5/6}$

2. If $y = \ln \frac{(x^2 + 1)^5}{\sqrt{1 - x}}$, then $\frac{dy}{dx}$ =

(A) $\frac{5}{x^2 + 1} - \frac{1}{2(1 - x)}$

(B) $\frac{\sqrt{1 - x}}{(x^2 + 1)^5}$

(C) $\frac{10x}{x^2 + 1} - \frac{1}{2(1 - x)}$

(D) $\frac{10x}{x^2 + 1} + \frac{1}{2(1 - x)}$

(E) $\frac{5}{x^2 + 1} + \frac{1}{2(1 - x)}$

3. The absolute maximum value of $y = 3x - e^x$ is

(A) $3 - e$ (B) $3 \ln 3 - 3$ (C) $9 - e^3$
(D) does not exist (E) -1

4. If $3 + \ln a = 2 \ln b + \ln 5$, then $a =$

(A) $b^2 + 5 - e^3$ (B) $b^2 + \frac{5}{e^3}$ (C) $\frac{5\,b^2}{e^3}$
(D) $e^{-3}(b^2 + 5)$ (E) $2b + 5 - e^3$

5. If $f(x) = x^x$, then $f\,'(e) =$

(A) e^{e+1} (B) e^e (C) e^{e-1} (D) 1 (E) $2e^e$

6. $\displaystyle\int_{0}^{\ln 4} \frac{2e^t}{1 + 2e^t}\, dt =$

(A) $\ln 3$ (B) $2 \ln \left(\frac{1}{2}\right)$ (C) $\ln 4$ (D) $\ln 6$ (E) $\ln \left(\frac{2}{3}\right)$

7. The ozone in the upper atmosphere is being gradually destroyed by the release of chlorofluorocarbons from refrigerators, air conditioners, hair sprays, etc., at a continuous yearly rate of 0.25%. At this rate, the number of years it will take for half the ozone to disappear is

(A) $\frac{1}{2} \ln 400$ (B) $\ln 200$ (C) 200

(D) $2 \ln 400$ (E) $400 \ln 2$

8. According to a graph in the December 30 issue of Time Magazine, the number of people in the world infected with HIV (the AIDS virus) was about 7.5 million in 1988, 22.5 million in 1996, thus tripling in 8 years. If the number is assumed to be rising exponentially, the number of people infected with HIV in the year 2000 will be (in millions) about

(A) 30 (B) $(22.5)\,\frac{3}{2}$ (C) $(7.5)\,3^{3/2}$

(D) 45 (E) $(7.5)\ln 3$

9. $\lim\limits_{x \varnothing 0} (e^x + 3x)^{\frac{1}{x}} =$

(A) 1 (B) e^3 (C) e (D) ∞ (E) e^4

10. $\lim_{t \to 0} \dfrac{t \sin t}{1 - \cos t} =$

(A) $\dfrac{3}{2}$ (B) 2 (C) ∞ (D) $\dfrac{\pi}{2}$ (E) 0

11. The function $f(x) = \dfrac{2x - 1}{x - 1}$, $x > 1$, is a decreasing function. If g is the inverse function to f, then $g(x) =$

(A) $\dfrac{x + 1}{2x + 1}$ (B) $\dfrac{x - 1}{x - 2}$ (C) $\dfrac{x - 2}{x - 1}$

(D) $\dfrac{2x + 1}{x + 1}$ (E) $\dfrac{x - 1}{2x - 1}$

12. The function $f(x) = e^x(x - 1)$ is an increasing function for $x \geq 0$. If g is the inverse function to f, then the derivative $g'(0)$ =

(A) e (B) $\frac{1}{e}$ (C) 0

(D) does not exist (E) 1

13. $\sin\left(\tan^{-1}\frac{2}{3}\right)$ =

(A) $\frac{3}{\sqrt{13}}$ (B) $\frac{2}{\sqrt{5}}$ (C) $\frac{2}{\sqrt{13}}$ (D) $\frac{\sqrt{13}}{3}$ (E) $\frac{\sqrt{5}}{3}$

14. If $f(x) = \tan^{-1}\left(\sqrt{x}\right)$, then $f'(4) =$

(A) $\frac{1}{5}$ (B) $-\frac{1}{4\tan^2 2}$ (C) $-\frac{1}{\tan^2 2}$

(D) $\frac{1}{4\sqrt{3}}$ (E) $\frac{1}{20}$

15. $\displaystyle\int_0^{2/3} \frac{1}{4 + 9x^2}\, dx =$

(A) $\frac{\pi}{12}$ (B) $\frac{\pi}{24}$ (C) $\frac{3\pi}{32}$ (D) $\frac{\pi}{16}$ (E) $\frac{\pi}{18}$

16. The slope of the curve $y = \cosh x$ at the point where $x = \ln 3$ is

(A) $\frac{5}{3}$ (B) $-\frac{5}{3}$ (C) $\frac{4}{3}$ (D) $-\frac{4}{3}$ (E) $\frac{8}{9}$

17. The solution of the initial value problem

$$\frac{dy}{dx} = xy^2, \quad y(1) = 2,$$

is $y =$

(A) $-\frac{2}{2 + x^2}$ (B) $\frac{2}{2 + x^2}$ (C) $\frac{2}{2 - x^2}$

(D) $\frac{2}{x^2 - 2}$ (E) $\frac{2}{x^2}$