1. The function $f(x) = e^x + x$ is an increasing function. If g is the inverse function to f, then the derivative $g'(1) =$

(A) $\frac{1}{2}$ (B) $\frac{1}{e+1}$

(C) $-\frac{1}{e+1}$ (D) -2 (E) 1

2. The slope of the curve $e^x \ln y = 1$ at the point $(0, e)$ is

(A) e (B) $-e$ (C) -1 (D) $\frac{1}{e}$ (E) 1

3. If $y = \sin^{-1}(\sqrt{x})$, then $\frac{dy}{dx} =$

(A) $\frac{1}{\sqrt{1-x}}$ (B) $\frac{1}{2\sqrt{x-x^2}}$ (C) $\sqrt{\frac{x}{1-x}}$

(D) $-\frac{1}{\cos(\sqrt{x})}$ (E) $-\csc(\sqrt{x})\cot(\sqrt{x})$

4. The population of a certain country is growing exponentially. In 1973 it had 17 million people, and in 1985 it had 19 million people. In the year 2000 the population (in millions) should be

(A) $\frac{19(\ln 27 - \ln 12)}{17}$ (B) $\frac{19}{17}e^{27/12}$

(C) $\frac{27(\ln 19 - \ln 17)}{15}$ (D) $\frac{43}{2}$ (E) $\frac{19^{9/4}}{17^{5/4}}$

5. $\lim_{x \varnothing 0} \dfrac{x \sin x}{\ln(\cos x)} =$

(A) -2 (B) 1 (C) 0 (D) $\dfrac{1}{2}$ (E) ∞

6. $\displaystyle\int_0^1 (5x^4 + 3x^2) \tan^{-1} x \, dx =$

(A) $\dfrac{\pi}{4} + \dfrac{1}{4}$ (B) $\dfrac{\pi}{6} + \dfrac{3}{4}$ (C) $\dfrac{\pi}{4}$

(D) $\dfrac{\pi}{2} - \dfrac{1}{4}$ (E) $\dfrac{\pi}{2}$

7. $\displaystyle\int_0^{\pi/2} \sin^2 x \cos^3 x \, dx =$

(A) $\frac{1}{3}$ (B) $\frac{5}{18}$ (C) $\frac{3}{4}$ (D) $\frac{2}{15}$ (E) $\frac{1}{6}$

8. The solution of the initial value problem

$$\frac{dy}{dx} - y = e^x, \quad y(0) = 2,$$

is $y =$

(A) $(x + 2)e^x$ (B) $\frac{1}{2} e^x + \frac{3}{2} e^{-x}$ (C) $xe^x + 2$

(D) $\frac{1}{2} e^x + \frac{3}{2}$ (E) $x + 2e^x$

9. $\displaystyle\int \frac{dx}{\sqrt{2x - x^2}} =$

(A) $\sinh^{-1}(x + 1) + C$

(B) $2\sec^{-1}\left(\frac{\sqrt{x}}{2}\right) + C$

(C) $\sin^{-1}(x - 1) + C$

(D) $\dfrac{\sqrt{2x - x^2}}{1 - x} + C$

(E) $2\sqrt{2x - x^2} + C$

10. $\displaystyle\int_1^3 \frac{5x + 6}{x^2 + 2x}\, dx =$

(A) $6 \ln 3 - 3 \ln 5$

(B) $4 \ln 5 - \ln 3$

(C) $4 \ln 3 - \ln 5$

(D) $2 \ln 3 + \ln 5$

(E) $\ln 3 + 2 \ln 5$

11. $\int_{1}^{\infty} \left(\frac{1}{x^2} - e^{-x} \right) dx =$

(A) $-1 + \frac{1}{e}$ (B) diverges (C) $-1 - \frac{1}{e}$

(D) $1 - \frac{1}{e}$ (E) $1 + \frac{1}{e}$

12. $\int_{-2}^{4} \frac{dx}{x} =$

(A) ln 3 (B) ln 8 (C) diverges

(D) ln 6 (E) ln 2

13. Given the series $\sum_{n=1}^{\infty} \frac{n!(n+2)}{(n+1)!}$,

(A) the Ratio Test shows that the series converges

(B) the Ratio Test shows that the series diverges

(C) the Comparison Test shows that the series converges

(D) the n–th Term Test shows that the series diverges

(E) the Integral Test shows that the series converges

14. Given the series $\sum_{n=1}^{\infty} \frac{n^8}{2^n}$,

(A) the Comparison Test shows that the series diverges

(B) the Ratio Test shows that the series converges

(C) the Ratio Test shows that the series diverges

(D) the n-th Root Test shows that the series diverges

(E) the n–th Term Test shows that the series converges

15. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2 + n}$

(A) diverges, by the Comparison Test

(B) diverges, by the Integral Test

(C) diverges, by the Ratio Test

(D) converges conditionally

(E) converges absolutely

16. The radius of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(-2)^n n^2 (x+2)^n}{2n+1}$$

is

(A) 4 (B) $\frac{1}{2}$ (C) ∞ (D) 0 (E) 2

17. The degree 3 term of the Maclaurin series for $\ln(2 + x)$ is

(A) $\frac{1}{12}\ x^3$ (B) $\frac{1}{48}\ x^3$ (C) $\frac{1}{6}\ x^3$

(D) $\frac{1}{3}\ x^3$ (E) $\frac{1}{24}\ x^3$

18. The degree 2 term of the Maclaurin series for $\frac{e^x}{1 - x}$ is

(A) $\frac{3}{2}\ x^2$ (B) x^2 (C) $\frac{1}{2}\ x^2$ (D) $\frac{5}{2}\ x^2$ (E) $2x^2$

19. The 3[rd] order Taylor polynomial for $f(x) = \frac{x}{\sqrt{1 + x}}$ at $a = 0$ is

(A) $\frac{1}{2}x - x^2 + \frac{3}{4}x^3$

(B) $2x - x^2 - \frac{1}{2}x^3$

(C) $x - \frac{1}{2}x^2 + \frac{3}{8}x^3$

(D) $1 + \frac{1}{2}x + \frac{3}{8}x^2 - \frac{5}{48}x^3$

(E) $x + \frac{1}{2}x^2 - \frac{3}{8}x^3$

20. If $y = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)^2} = x - \frac{x^3}{3^2} + \frac{x^5}{5^2} - \frac{x^7}{7^2} + \ldots$, then $\frac{dy}{dx} =$

(A) $\frac{1}{1 + \frac{1}{3}x^2}$

(B) $\frac{\sin x}{x}$

(C) $\cos\left(\frac{x}{\sqrt{3}}\right)$

(D) $\frac{\tan^{-1} x}{x}$

(E) $\ln(1 + x^2)$

21. The parametric equations

$$x = \sqrt{3 + 2t} \quad , \quad y = \sqrt{2 - 3t} \quad , \quad -\frac{3}{2} \le t \le \frac{2}{3} \quad ,$$

represent part of

(A) a circle
(B) an ellipse with a focus on the y-axis
(C) an ellipse with a focus on the x-axis
(D) a hyperbola with a focus on the y-axis
(E) a hyperbola with a focus on the x-axis

22. The polar equation $r \cos \theta = 6 - \frac{3}{2} r$ represents

(A) a parabola
(B) an ellipse with eccentricity $\frac{2}{3}$
(C) an ellipse with eccentricity $\frac{3}{2}$
(D) a hyperbola with eccentricity $\frac{2}{3}$
(E) a hyperbola with eccentricity $\frac{3}{2}$

23. A surface is obtained by rotating the curve

$x = 3 \cos t + 4 \sin t,$

$y = 4 \cos t - 3 \sin t + 5 ,$

$0 \le t \le \pi$

about the x-axis. The area of the surface is

(A) 60π (B) $10\pi (6\pi - 5)$ (C) $10\pi(5\pi - 6)$

(D) 80π (E) $2\pi(8\pi - 3)$

24. The polar equation $r = \dfrac{\sec\theta\tan\theta}{1 + \tan^3\theta}$

represents a curve whose Cartesian equation is

(A) $x^2 + xy^2 = xy$ (B) $x^3 + xy^2 = y$ (C) $x^3 + y = x^2y$

(D) $x^3 + y^3 = xy$ (E) $x + y^3 = y$

25. A curve is given by a polar equation

$$r = e^{\theta},\ 0 \le \theta \le \pi.$$

The length of the curve is

(A) $2e^{2\pi}$ (B) $e^{2\pi} - 2$ (C) $\sqrt{2}\,(e^{\pi} - 1)$

(D) $2e^{\pi}$ (E) $2(e^{2\pi} - 1)$