1. Let $f(x) = x^3 + x + 2$. It can (easily) be shown that $f(x)$ has an inverse function $g(x)$. Find $g'(4)$.

(A) $\frac{2}{3}$ (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) 1 (E) $\frac{1}{3}$

2. How many of the following are true?

i) $\int_1^{\frac{1}{e}} \frac{1}{x}\, dx = -1$

ii) $\frac{d}{dx}(2^x) = x2^{x-1}$

iii) $\frac{d}{dx}(\log_{3/2} x) = \frac{1}{x(\ln 3 - \ln 2)}$

iv) $e^{x+2y} = e^x + 2e^y$

(A) none (B) two (C) one (D) four (E) three

3. Let $f(x) = \sqrt[3]{(x+3)^2 (x-1)}$. The slope of the graph of f at $x = -1$ is

(A) $\frac{2}{3}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{\sqrt[3]{2}}$ (D) $-\frac{1}{3}$ (E) 1

4. Suppose a tree grows at a yearly rate equal to $\frac{1}{10}$ of its height. If the tree is 10 ft. tall now, approximately how tall will it be in 5 years?

(A) 20 ft (B) 12 ft (C) 16 ft (D) 25 ft (E) 50 ft

5. $$\lim_{x \to 1} \frac{\frac{x-3}{4} + \frac{1}{x+1}}{(x-1)^2} = ?$$

(A) $-\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $-\frac{2}{3}$ (D) $\frac{1}{8}$ (E) ∞

6. $$\lim_{x \to \infty} \left(\frac{x}{x-3}\right)^{2x} = ?$$

(A) e^2 (B) $e^{-2/3}$ (C) e^{-3} (D) 1 (E) e^6

7. $\dfrac{1 + \tanh x}{1 - \tanh x} = ?$

(A) e^{2x} (B) $\cosh x + \sinh x$ (C) $\dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

(D) $\operatorname{sech} x + \operatorname{csch} x$ (E) $1 - 2e^x$

8. If $f(x) = \arcsin\left(e^x\right)$, then $f'(x) = ?$

(A) $\dfrac{1}{1 + e^{2x}}$ (B) $\dfrac{e^x}{1 + e^{2x}}$ (C) $\dfrac{1}{\sqrt{1 - e^{2x}}}$

(D) $\dfrac{1}{e^x\sqrt{e^{2x} - 1}}$ (E) $\dfrac{e^x}{\sqrt{1 - e^{2x}}}$

9. Find the equation of the curve that passes through the point (0,1) and whose slope at (x,y) is $\frac{y^2}{\sec x}$.

(A) $y = 1 + \sin x$

(B) $y = \frac{1}{2 - \cos x}$

(C) $y = \frac{\cos x}{1 + \sin x}$

(D) $y = \frac{1}{1 - \sin x}$

(E) $y = \frac{1 - \sin x}{\cos x}$

10. The solution of the initial value problem

$$x \frac{dy}{dx} + 2y = x^2 + 1, \quad x > 0 \quad \text{and} \quad y(1) = 1$$

is $y = ?$

(A) $\frac{x^2}{2} + x + 1 - \frac{1}{2x^2}$

(B) $\frac{x^2}{4} + \frac{1}{2x^2}$

(C) $\frac{x}{4} + 1 - \frac{2}{x}$

(D) $\frac{x^4}{4} + \frac{x^2}{2} + 1 - \frac{1}{x^2}$

(E) $\frac{x^2}{4} + \frac{1}{2} + \frac{1}{4x^2}$

11. In the partial fraction decomposition of $\frac{2x}{(x + 1)(x^2 + 1)}$, the numerator whose denominator is $x^2 + 1$ is

(A) 1 (B) $2x$ (C) $x + 1$ (D) $3x - 1$ (E) $2 - x$

12. If the standard trigonimetric substitution is made, the integral

$$\int \frac{\sqrt{x^2 - 4}}{x^4}\, dx \quad \text{becomes}$$

(A) $2\int \frac{\sin\theta}{\cos^4\theta}\, d\theta$

(B) $\frac{1}{4}\int \frac{\tan^2\theta}{\sec^3\theta}\, d\theta$

(C) $\frac{1}{8}\int \tan\theta \sec^3\theta\, d\theta$

(D) $4\int \sin^2\theta \cos^2\theta\, d\theta$

(E) $\frac{1}{4}\int \frac{\sec^2\theta}{\tan^4\theta}\, d\theta$

13. $\int_{1}^{e} x \ln x \, dx = ?$

(A) $\frac{1}{2}(e - 1)$ (B) $\frac{1}{4}(3e^2 - 1)$ (C) $\frac{1}{4}$

(D) $\frac{1}{2}(e^2 - 1)$ (E) $\frac{1}{4}(e^2 + 1)$

14. $\int_{0}^{3} \frac{1}{(x-1)^3} \, dx$?

(A) $\frac{1}{2}$ (B) $\frac{3}{8}$ (C) diverges (D) $\frac{1}{6}$ (E) $\frac{1}{4}$

15. $\int_{0}^{\infty} \left(\pi - \frac{2x}{1 + x^2} - 2 \arctan x \right) dx = ?$

(A) 2 (B) $\frac{3\pi}{4}$ (C) ln 2 (D) π (E) diverges

16. How many of the following infinite series are convergent?

$$\sum_{n=1}^{\infty} \frac{n^5}{5^n}, \quad \sum_{n=1}^{\infty} \frac{n!}{(n+2)!}, \quad \sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}, \quad \sum_{n=1}^{\infty} \frac{n^2+1}{n^3+1}$$

(A) one (B) four (C) none (D) two (E) three

17. The series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin n}{n^2}$

(A) diverges, by the Ratio Test.
(B) diverges, by the nth Term Test.
(C) converges absolutely.
(D) diverges by the Integral Test.
(E) converges conditionally.

18. The third order Taylor polynominal at $a = 0$ for the function

$$f(x) = e^{-2x} \cos x$$

is equal to

(A) $1 - 2x + \frac{3}{2}x^2 - \frac{1}{3}x^3$

(B) $1 + x - \frac{1}{2}x^2 + \frac{2}{3}x^3$

(C) $1 - 2x - 2x^2 + \frac{4}{3}x^3$

(D) $1 - x - \frac{3}{2}x^2 - \frac{13}{6}x^3$

(E) $1 + 2x - \frac{1}{2}x^2 + \frac{5}{6}x^3$

19. Which of the following series have radius of convergence greater than 1?

$\sum_{n=1}^{\infty} \frac{5^n}{n^2} x^n$, $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n \cdot 3^n}$, $\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{n!}$, $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n+1}$

(A) the first 2 (B) the middle 2 (C) the last 3

(D) only the 3^{rd} one (E) all of them

20. A set of polar coordinates for the point whose Cartesian coordinates are $(-2\sqrt{3}, 2)$ is

(A) $\left(4, \frac{7}{6}\pi\right)$ (B) $\left(-4, \frac{5}{6}\pi\right)$ (C) $\left(4, \frac{2\pi}{3}\right)$

(D) $\left(-4, -\frac{\pi}{6}\right)$ (E) $\left(4, \frac{7}{3}\pi\right)$

21. The polar equation of the circle of radius 13 whose center has Cartesian coordinates (12, –5) is given by

(A) $r = 13 \sin \theta$
(B) $r^2 = 5 \sin \theta - 12 \cos \theta$
(C) $r = 2\sqrt{3} \cos \theta - \sqrt{5} \sin \theta$
(D) $r^2 = 12 \cos \theta - 5 \sin \theta$
(E) $r = 24 \cos \theta - 10 \sin \theta$

22. The polar equation $r = \sin \theta \tan \theta$ represents a curve whose Cartesian equation is

(A) $x^3 + xy^2 = y^2$

(B) $x^2 + y^2 = xy$

(C) $x^2y^2 + x^4 = y^3$

(D) $\sqrt{x^2 + y^2} = x$

(E) $x\sqrt{x^2 + y^2} = y$

23. The graph of the polar equation

$$r^2 = 4 \cos 2\theta$$

most closely represents

(A)

(B)

(C)

(D)

(E)

24. The area inside one loop of the curve $r = 6 \sin 3\theta$ is

(A) 6π (B) $2\sqrt{3}\ \pi$ (C) 3π (D) $\frac{9}{2}\ \pi$ (E) 2π

25. A curve is given by the polar equation

$$r = \sin^2\left(\frac{\theta}{2}\right), \quad 0 \le \theta \le \pi .$$

The length of the curve is given by

(A) $\frac{2}{3}\ \pi$ (B) 2 (C) $\pi - 1$ (D) $\frac{\pi}{2}$ (E) $1 + \sqrt{2}$

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Virgil