

1. It is easy to show that the function $y = \frac{5x + 3}{x - 4}$ has an inverse. What is it?

(A) $y = \frac{x - 4}{5x + 3}$ (B) $y = \frac{4x - 1}{3x + 5}$ (C) $y = \frac{3x - 5}{4x + 1}$

(D) $y = \frac{4x + 3}{x - 5}$ (E) $y = \frac{5x - 3}{x + 4}$

2. Suppose that f and g are increasing functions with domain and range $(-\infty, \infty)$. How many of the following functions must be one-to-one?

i) $f + g$ ii) fg iii) $\frac{f}{g}$ iv) $f \circ g$

(A) 4 (B) 1 (C) 2 (D) 0 (E) 3

3. It is easy to see that the function $f(x) = x + \sqrt{x}$ has an inverse function $g(x)$. Find $g'(2)$.

(A) $\frac{2}{3}$ (B) $\frac{1}{1 + 2\sqrt{2}}$ (C) $\frac{1}{2}$ (D) $\frac{2\sqrt{2}}{1 + 2\sqrt{2}}$ (E) $\frac{4}{5}$

4. How many of the following are true?

$$\text{i) } \ln \frac{1}{3} = \int_3^1 \frac{1}{t} dt$$

$$\text{ii) } e^{-x} < 0 \text{ for all } x$$

$$\text{iii) } 2^\pi = e^{2 \ln \pi}$$

$$\text{iv) } \log_4 x = 2 \log_2 x$$

- (A) 0 (B) 3 (C) 2 (D) 1 (E) 4

5. If $y = \sqrt{\frac{(x^4 + 1)(x^3 + 1)}{(x^2 + 1)(x + 1)}}$, then $\left. \frac{dy}{dx} \right|_{x=1} = ?$

- (A) $\sqrt{6}$ (B) 1 (C) $\sqrt{\frac{7}{3}}$ (D) $\frac{1}{2}$ (E) $\sqrt{3}$

6. Suppose the populations of two countries are growing exponentially. Suppose also that one country has a population of 60,000,000 and a doubling time of

20 years whereas the other has a population of 20,000,000 and a doubling time of 10 years. Approximately how long (in years) will it be until the two countries have the same population?

- (A) never (B) $10 \frac{\ln(\frac{3}{2})}{\ln 2}$ (C) $\frac{3}{2} \ln 2$
- (D) $2 \ln(\frac{3}{2})$ (E) $20 \frac{\ln 3}{\ln 2}$

7. $\lim_{x \rightarrow 1} \frac{\ln x - x + 1}{x^3 - 3x + 2} = ?$

- (A) $-\frac{1}{6}$ (B) 0 (C) $-\frac{1}{2}$ (D) $\frac{1}{3}$ (E) 1

8. If $f(x) = x^{\ln x}$, then $f'(e) = ?$

- (A) 1 (B) $\frac{1}{e}$ (C) 2 (D) e (E) 0

9. $\int_0^2 \frac{1}{1 + (x - 1)^2} dx = ?$

- (A) 1 (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{2}$ (E) $\frac{\pi}{2}$

10. The region between the curve $y = \sqrt{\cot x}$ and the x -axis from $x = \frac{\pi}{6}$ to $x = \frac{\pi}{2}$ is revolved about the x -axis to generate a solid.

The volume of the solid is

- (A) $2\pi\sqrt{3}$ (B) $\pi \ln 2$ (C) πe^2 (D) $\pi\sqrt{2}$ (E) 3π

11. A ladder 10ft. long leans against a vertical wall. If the bottom of the ladder slides away from the base of the wall at a speed of 2ft/sec, how fast is the angle between the ladder and the wall changing (in radians/sec) when the bottom of the ladder is 6 ft. from the base of the wall?

- (A) $\frac{1}{4}$ (B) $\frac{3}{5}$ (C) $\frac{5}{6}$ (D) $\frac{1}{5}$ (E) $\frac{1}{3}$