

1. $\lim_{x \rightarrow \infty} \tanh x = ?$

- (A) e^2 (B) 0 (C) ∞ (D) $\frac{1}{2e}$ (E) 1

2. How many local maxima does the function $y = \sinh x - (x - 1) \cosh x$ have?

- (A) 2 (B) 1 (C) none (D) 4 (E) infinitely many

3. Solve the following initial value problem for y as a function of x :

$$x \frac{dy}{dx} + 2y = x^2 + 1, \quad x > 0 \text{ \& } y(1) = 1$$

(A) $y = \frac{x}{2} + \frac{1}{2x}$

(B) $y = e^{x^2} + 2e^x - 2$

(C) $y = \frac{x^2}{4} + \frac{1}{2} + \frac{1}{4x^2}$

(D) $y = \ln x + \frac{1}{x}$

(E) $y = \frac{x^3}{6} + \frac{2}{3} + \frac{1}{6x^3}$

4. The solution of the initial value problem

$$\frac{dy}{dx} = y^2 \cos x, \quad y(0) = 1$$

is $y =$

(A) $\frac{1}{1 - \sin x}$

(B) $\frac{\cos x}{1 + x^2}$

(C) $e^x + \sin x$

(D) $\sec x$

(E) $x \sin x + 1$

5. $\int x \cos 2x \, dx = ?$

(A) $\frac{x^2 \sin 2x}{4} + C$

(B) $\frac{1}{2} (\sin 2x - \cos 2x) + C$

(C) $\frac{x}{2} (\sin 2x + \cos 2x) + C$

(D) $\frac{x \sin 2x}{2} + \frac{\cos 2x}{4} + C$

(E) $\frac{1 - \cos 4x}{2} + C$

6. $\int_0^1 \arctan x \, dx = ?$

(A) $\sqrt{3} - \frac{\pi}{4}$

(B) $\frac{1}{2}$

(C) $\frac{\pi}{4} - \frac{1}{2} \ln 2$

(D) $\frac{\sqrt{3}}{2}$

(E) $\frac{\pi}{3} + \ln 3$

7. $\int_2^3 \frac{x^3 - 1}{x^3 - x} dx = ?$

(A) $\frac{2}{3}$

(B) $\pi - 2\ln 3$

(C) $\frac{1}{2} + \frac{\pi}{6}$

(D) $1 + \ln \frac{9}{8}$

(E) $\sqrt{3} - \frac{2}{3}$

8. In the partial fraction decomposition of

$$\frac{3x^2 + x + 2}{x^2(x^2 + 1)},$$

the numerator of the term whose denominator is $x^2 + 1$ is

(A) $1 - x$

(B) 1 (C) $2x + 1$

(D) 2

(E) x

9. $\int \frac{1}{x^2 \sqrt{x^2 - 4}} dx = ?$

(A) $2\sqrt{x^2 - 4} - \frac{1}{4} \arcsin\left(\frac{x}{2}\right) + C$

(B) $\frac{x}{2}\sqrt{x^2 - 4} + C$

(C) $\frac{1}{2} \ln(x^2 - 4) + \frac{x}{2} + C$

(D) $-\frac{x^2}{(x^2 - 4)^{3/2}} + C$

(E) $\frac{\sqrt{x^2 - 4}}{4x} + C$

10. To find the integral $\int \sqrt{7 - 6x - x^2} dx$,

the method of trigonometric substitution can be used. A suitable substitution is

(A) $x = 4 \sec \theta + 3$

(B) $x = 4 \sin \theta - 3$

(C) $x = 4 \tan \theta - 3$

(D) $x = 4 \sin \theta + 3$

(E) $x = 4 \tan \theta + 3$