Mathematics 126: Test I

September 22, 1998

November 8, 2004

Name: _____

- The test runs for one hour.

- All work is to be your own.

- You may use one sheet of paper with formulae and notes, but should not use any other notes or your book.

- You may use graphing calculators (but not palmtops or laptops). You may not use

  1. notes in your calculator;
  2. numerical integration facilities on your calculator; or
  3. algebra, differentiation, or integration facilities of your calculator.

The possible answers to the seven multiple choice questions are either correct or wrong. *Please write the letter of the correct answer in the space provided below.* The remaining questions will require that you demonstrate knowledge of a correct procedure for arriving at the answer. Please circle the answer you want us to consider. The correct answer is not enough for full credit: you must indicate how you arrived at it. Please mark out any work on a problem that you do not want us to consider.

Problem	Answer	Possible Points	Points Lost
1		7	
2		7	
3		7	
4		7	
5		7	
6		7	
7		7	
8	———	12	
9	———	12	
10	———	12	
11	———	15	

Total Points Lost _____

Problem 1 (7 points total) $f(x) = x^5 + x + 1$ *is a differentiable increasing function defined* $(-\infty, \infty)$. *Let* $g(x)$ *denote the inverse of* $f(x)$. *Noting that* $f(0) = 1$, *compute* $g'(1)$.

a)   3   b)   6   c) $\dfrac{1}{6}$ d) $\dfrac{1}{3}$ e)   1

Problem 2 (7 points total) *Find* $\displaystyle\lim_{x\to\infty} x^{\frac{1}{\ln(x+2)}}$.

a)   0   b) e c) $e^{\frac{1}{2}}$ d)   1   e) $\dfrac{1}{2}$

Problem 3 (7 points total) *Charcoal caused by a volcanic eruption has 25% of the carbon-14 found in living matter. About how long ago was the volcanic eruption. Assume that the half life of carbon-14 is 5,700 years. You may use $\ln 2 = 0.69$ and $\ln 0.25 = -1.38$.*

a)   14,250   b)   2,850   c)   5,700   d)   8,550   e)   11,400

Problem 4 (7 points total) *For x satisfying $|x| < 1$, $\tan(\sin^{-1}(x)) = $*

a) $\dfrac{1 - x^2}{x}$ b) $\sqrt{1 - x^2}$ c) $\dfrac{x}{\sqrt{1 - x^2}}$ d) $\dfrac{1}{1 + x^2}$ e) $\dfrac{x}{1 + x^2}$

Problem 5 (7 points total) $\int_0^4 \dfrac{2x\mathrm{d}x}{x^2 + 2} =$

a) $\ln 16$ b) $\ln 36$ c) $2(\tan^{-1}(18) - \tan^{-1}(2))$ d) $\sec^{-1}(18) - \sec^{-1}(2)$ e) $2\ln 3$

Problem 6 (7 points total) *Use Euler's method with* dx=0.5 *to estimate* $y(2.5)$ *if* $y' = 1 - y/x$ *and* $y(2) = -1$.

a) 0.7   b) 0.75   c) −0.25   d) 0.25   e) 0.5

Problem 7 (7 points total) *Which of the following functions satisfies the initial value problem* $x(1+x^2)y' = \tan(y)$ *with* $y(0) = 0$.

a) $\dfrac{1}{1+x^2}$
b) $\tan^{-1}(x)$
c) $\tan^{-1}(x^2)$
d) x
e) $\tan^{-1}(x) + 1$

Problem 8 (12 points total) $\displaystyle \int \sqrt{1 + \sinh^2(x)}\,\mathrm{d}x =$

Problem 9 (12 points total) $\displaystyle\int \frac{\mathrm{d}x}{4x^2 + 16x + 17} =$

Problem 10 (12 points total) $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{4 - x^2}} =$

Problem 11 (15 points total) *Solve the differential equation* $y' + \sin(x)y = 2xe^{\cos(x)}$.