1. For $x > 0$, the maximum value of $y = \frac{\ln x}{x^2}$ is

(A) $\frac{1}{2e}$ (B) $\frac{1}{e^2}$ (C) $\frac{2}{e}$ (D) $\frac{\ln 2}{4}$ (E) $4 \ln \left(\frac{1}{2}\right)$

2. The region bounded by the curve $y = \frac{1}{x^2 + 1}$, the lines $x = 1$ and $x = 3$, and the x-axis, is revolved about the y-axis. The volume of the solid obtained in this way is

(A) $\pi \ln 10$ (B) $2\pi \left(\tan^{-1} 5 - \tan^{-1} 1\right)$

(C) $\pi \ln 5$ (D) $\pi \ln 13$

(E) $2\pi \left(\tan^{-1} 3 - \tan^{-1} 1\right)$

3. The tangent line to the curve $y = x^2 e^{2x}$ at the point $(1, e^2)$ has slope

(A) $5e^2$ (B) $4e^2$ (C) e (D) $4e^2 - 1$ (E) $2e$

4. If $\frac{2^{3a}}{4} = 3^{2a+1}$, then $a =$

(A) $\frac{2 \ln 2 + \ln 3}{3 \ln 2 - 2 \ln 3}$ (B) $\frac{e^4}{2} - \frac{e}{3}$ (C) $\frac{3 \ln 2 + 2 \ln 3}{2 \ln 2 - \ln 3}$

(D) $e^{3/4} - 3e^2$ (E) $\frac{\ln 2 + 2 \ln 3}{3 \ln 2 - 2 \ln 3}$

5. $\int_{1}^{\sqrt{2}} x\, 2^{(x^2)}\, dx =$

(A) $\frac{1}{\ln 2}$ (B) $\frac{3}{\ln 2}$ (C) $\frac{1}{3 \ln 2}$ (D) $\ln 2$ (E) $3 \ln 2$

6. $\int_{0}^{\ln \pi} e^x \sin(e^x)\, dx =$

(A) $\pi + \cos(e^\pi)$ (B) $e^\pi + \cos 1$ (C) $1 + \cos e$

(D) $1 + \cos 1$ (E) $\cos(e^\pi)$

7. The number of influenza cases in the U.S. on January 10 was estimated to be 2200, while the number on January 16 was estimated to be 11000. If the number of cases is assumed to be growing exponentially, then the number of cases on January 19 would be about

(A) 32000 (B) 24200 (C) 26400
(D) 15400 (E) 16800

[You may use any of the approximations $\ln 5 = 1.6$, $\sqrt{5} = 2.2$, $e^{2.4} = 11$]

8. A certain radioactive element takes 450 years to decay to $\frac{3}{4}$ of its original amount. Its half-life (in years) is

(A) $\dfrac{450 \ln 2}{\ln 3 - \ln 2}$ (B) 675 (C) $\dfrac{450 \ln 2}{2 \ln 2 - \ln 3}$

(D) 900 (E) $450 \ln\left(\frac{3}{2}\right)$

9. $\lim_{x \to 0} \dfrac{1 - \cos x}{\ln(1 + x^2)} =$

(A) ∞ (B) $\frac{1}{2}$ (C) 1 (D) 0 (E) 2

10. For $x \leq 0$, the function $f(x) = x^2 - x$ is a decreasing function. If g is the inverse function to f, then $g(6) =$

(A) 3 (B) 1 (C) -2 (D) -1 (E) -3

11. Suppose f is an increasing function, and g is its inverse function. If

$$f(1) = 2, \quad f(2) = 3, \quad f'(1) = 3, \quad f'(2) = 4$$

then one of the following is <u>true</u>

(A) $g'(2) = \frac{1}{3}$ (B) $g(3) = 1$ (C) $g'(2) = \frac{1}{4}$

(D) $g(2) = 3$ (E) $g'(3) = \frac{1}{3}$

12. $\sin^{-1}\left(\cos \frac{2\pi}{3}\right) =$

(A) $\frac{\pi}{4}$ (B) $-\frac{2\pi}{3}$ (C) $-\frac{\pi}{3}$ (D) $-\frac{\pi}{6}$ (E) $\frac{\pi}{6}$

13. If $y = \sin^{-1}(x^2 - 1)$, $0 < x < 1$, then $\frac{dy}{dx}$ =

(A) $\frac{2}{\sqrt{2 - x^2}}$ (B) $x\sqrt{2 - x^2}$ (C) $\frac{2x}{x^4 - 2x^2 + 2}$

(D) $\frac{2}{\sqrt{1 - x^2}}$ (E) $2x\sqrt{1 - x^2}$

14. $\displaystyle\int_{\frac{1}{2}}^{\frac{\sqrt{2}}{2}} \frac{dx}{\sqrt{1 - x^2}\ \sin^{-1} x}$ =

(A) $\ln 2$ (B) $\ln\left(\frac{4}{3}\right)$ (C) $\ln\left(\frac{\pi}{8}\right)$

(D) $\ln\left(\frac{3}{2}\right)$ (E) $\ln\left(\frac{\pi}{12}\right)$

15. $\int_{0}^{\ln 2} \sinh x \, dx =$

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$ (E) $\frac{2}{3}$

16. The solution of the initial value problem

$$\frac{dy}{dx} + 2xy = e^{-x^2}, \ y(2) = 0,$$

is $y =$

(A) $(x + 2)e^{x^2}$ (B) $xe^{x^2} + 2$ (C) $(x - 2)\, e^{-x^2}$

(D) $(x + 2)e^{-x^2}$ (E) $(x - 2)e^{x^2}$

17. $\int \frac{dt}{\sqrt{-t^2 + 4t - 3}} =$

(A) $\tan^{-1}(t - 2) + C$

(B) $\tan^{-1}(t + 2) + C$

(C) $\sec^{-1}(t - 2) + C$

(D) $\sin^{-1}(t - 2) + C$

(E) $\sin^{-1}(t + 2) + C$