1. One of the following is false (as x ∅ ∞)

(A) $e^{2x} = o(e^{3x})$ (B) $x^2 = o(x \ln x)$ (C) $x = o(x^2 - \ln x)$

(D) $x^3 = o(e^x)$ (E) $\ln x = o(\sqrt{x})$

2. The region under the curve $y = \sqrt{x}\ e^x$, $0 \le x \le 1$, is rotated about the x-axis. The volume of the resulting solid is

(A) $\frac{\pi}{4}(3e^2 - 1)$ (B) $\pi(e - 1)$

(C) $\frac{\pi}{4}(e^2 - 1)$ (D) $\frac{\pi}{4}(e^2 + 1)$

(E) π

3. $\displaystyle\int_{0}^{\pi} x \sin x \, dx =$

(A) $\frac{2\pi}{3}$ (B) $\frac{1}{4}$ (C) π^2 (D) π (E) -2

4. $\displaystyle\int_{2}^{4} \frac{3x - 1}{x^3 - x} \, dx =$

(A) $\ln 2 + 3 \ln 3 - 2 \ln 5$

(B) $\ln 2 + 2 \ln 3 - \ln 5$

(C) $-\ln 2 + 3 \ln 3 - \ln 5$

(D) $2 \ln 2 - \ln 3 + \ln 5$

(E) $2 \ln 2 - 2 \ln 3 + \ln 5$

5. $\displaystyle\int_2^4 \frac{x^3 - x^2 + 2x - 1}{x^2 - x}\, dx =$

(A) $3 + 4 \ln\left(\frac{2}{3}\right)$

(B) $6 + \ln 6$

(C) $4 + \ln 6$

(D) $\ln 6$

(E) $4 + 3 \ln\left(\frac{3}{2}\right)$

6. To find the integral $\int \sqrt{3 + 2x - x^2}\, dx$, the method of trigonometric substitution can be used. A suitable substitution, and the resulting trigonometric integral, are

(A) $x = 2 \sin\theta + 1; \int 4 \cos^2\theta\, d\theta$

(B) $x = 2 \sin\theta + 1; \int 2 \cos\theta\, d\theta$

(C) $x = 2 \sin\theta - 1; \int 4 \cos^2\theta\, d\theta$

(D) $x = 2 \sin\theta - 1; \int 2 \cos\theta\, d\theta$

(E) $x = 2 \tan\theta + 1; \int 4 \sec^3\theta\, d\theta$

7. $\displaystyle\int_{0}^{2} \frac{dx}{(x-1)^2} =$

(A) $\frac{2}{3}$ (B) diverges (C) -2 (D) 0 (E) 2

8. $\displaystyle\int_{1}^{\infty} \frac{dx}{x^2+x} =$

(A) diverges (B) $\ln 2$ (C) $\frac{\pi}{4}$ (D) $\frac{1}{2} \ln 2$ (E) $\frac{1}{2}$

9. A sequence $\{a_n\}$ is given, with $a_1 = 1$, $a_2 = 1$, and with the recursion formula

$$a_{n+2} = a_{n+1} + \left((-1)^{a_n}\right)a_n.$$

Then, a_{11} =

(A) 1 (B) 7 (C) 0 (D) −1 (E) 8

10. $\lim_{n \varnothing \infty} \frac{(-1)^n \sin n}{n}$ =

(A) ∞ (B) 1 (C) does not exist (D) π (E) 0

11. $\lim\limits_{n \to \infty} \dfrac{\ln(n^2 + n)}{\ln(n + 1)} =$

(A) does not exist (B) ∞ (C) 0 (D) ln 2 (E) 2

12. The partial fraction expression for $\dfrac{2x + 1}{x^2(x + 1)^2}$ is

$$\frac{2x + 1}{x^2(x + 1)^2} = \frac{1}{x^2} - \frac{1}{(x + 1)^2} .$$

From this, one can deduce that

$$\sum_{n=1}^{\infty} \frac{2n + 1}{n^2(n + 1)^2} =$$

(A) diverges (B) 0 (C) $-\frac{1}{4}$ (D) $\frac{3}{4}$ (E) 1

13. Let $\sum_{n=1}^{\infty} a_n$ be an infinite series, with partial sums $s_1, s_2, s_3, \ldots$.

Which of the following statements is true?

(A) If $\sum_{n=1}^{\infty} a_n$ diverges, then $\lim_{n \varnothing \infty} a_n \neq 0$.

(B) If $a_n \geq 0$ for all n, then the series converges.

(C) If $\lim_{k \varnothing \infty} s_k = 0$, then $\lim_{n \varnothing \infty} a_n = 0$.

(D) If $\lim_{n \varnothing \infty} a_n = 0$, then $\lim_{k \varnothing \infty} s_k = 0$.

(E) If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{k \varnothing \infty} s_k = 0$.

14. $\sum_{n=0}^{k} 2^n =$

(A) $2^k + 2$ (B) $2^k + 1$ (C) $2^{k+1} + 1$

(D) $2^k - 1$ (E) $2^{k+1} - 1$

15. $\sum_{n=0}^{\infty} \left(\frac{3^{n-1}}{4^n} \right) =$

(A) $\frac{8}{5}$ (B) $\frac{10}{3}$ (C) diverges

(D) $\frac{4}{3}$ (E) $\frac{5}{6}$

16. Given the infinite series

(1) $\sum_{n=1}^{\infty} \left(\frac{3}{2} \right)^{n-1}$ (2) $\sum_{n=1}^{\infty} \frac{n^2}{n^2 + 1}$ (3) $\sum_{n=1}^{\infty} \left(\frac{2}{3} \right)^{n-1}$

(A) (1) converges, (2) diverges, (3) diverges

(B) (1) diverges, (2) converges, (3) converges

(C) (1) diverges, (2) diverges, (3) converges

(D) (1) converges, (2) diverges, (3) converges

(E) (1) diverges, (2) converges, (3) diverges