1. The function $f(x) = 2\sin^2 x + \sin x$, $0 \le x \le \frac{\pi}{2}$,

is an increasing function, so that the inverse function f^{-1} exists.
The value $f^{-1}(1)$ is equal to

(A) $-\frac{\pi}{2}$ (B) π (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$ (E) $\frac{\pi}{3}$

2. The slope of the curve $y = \ln(x - \sqrt{x} - 1)$ at the point $(4,0)$ is

(A) $\frac{1}{5}$ (B) 1 (C) $\frac{3}{4}$ (D) $\frac{1}{8}$ (E) $\frac{1}{4}$

3. If $y = \tan^{-1}\left(\sqrt{x^2 + 2x}\right)$, then $\frac{dy}{dx} =$

(A) $\frac{1}{\sqrt{2x + x^2}}$ (B) $\frac{1}{1 - x}$ (C) $\frac{1}{(x + 1)\sqrt{x^2 + 2x}}$

(D) $\frac{x + 1}{\sqrt{x^2 + 2x}}$ (E) $\frac{1}{(x + 1)^2}$

4. The population of a colony of bacteria is growing exponentially with time. It starts with 500 bacteria, and after 3 hours there are 8000 bacteria. The number of hours (after the beginning of the process) when the population reaches 30,000 is

(A) $\frac{15}{\ln 2}$ (B) $\frac{\ln 60}{16 \ln 3}$ (C) $\frac{\ln 60}{\ln 3}$

(D) $\frac{\ln 60}{\ln 2}$ (E) $\frac{3 \ln 60}{\ln 16}$

5. $\lim_{x \to 0} \dfrac{e^{2x} - 2x^2 - 2x - 1}{x^3} =$

(A) $\frac{8}{3}$ (B) $\frac{4}{3}$ (C) $\frac{5}{6}$ (D) 0 (E) $\frac{32}{3}$

6. $\displaystyle\int_0^1 (2x + 4x^3) \ln(1 + x^2)\, dx =$

(A) $2 \ln 2 - \frac{2}{5}$ (B) $\ln 2 + \frac{5}{8}$ (C) $\ln 2 + \frac{3}{5}$

(D) $6 \ln 2$ (E) $2 \ln 2 - \frac{1}{2}$

7. $\displaystyle\int_{-2}^{2} \frac{dx}{4 + 3x^2} =$

(A) $\frac{\pi}{4\sqrt{3}}$ (B) $\frac{\pi}{16}$ (C) $\frac{2\pi}{\sqrt{3}}$ (D) $\frac{2\pi}{3}$ (E) $\frac{\pi}{3\sqrt{3}}$

8. The solution of the initial value problem

$$\frac{dy}{dx} = \frac{2x^2 + 1}{xe^y}, \quad y(e) = 2,$$

is $y =$

(A) $e^x(x^2 + \ln x)$

(B) $\ln(x^2 + \ln x - 2)$

(C) $\ln\left(\frac{2}{3}x^3 + x - \frac{1}{2}x^2\right)$

(D) $\ln(x^2 + \ln x) + 2 - \ln(e^2 + 1)$

(E) $\ln(x^2 + \ln x - 1)$

9. $\displaystyle\int \frac{dx}{(1 + x^2)^{3/2}} =$

(A) $-\dfrac{4x}{\sqrt{1 + x^2}} + C$

(B) $\dfrac{x}{\sqrt{1 + x^2}} + C$

(C) $\dfrac{2}{5(1 + x^2)^{5/2}} + C$

(D) $\dfrac{1}{3x\sqrt{1 + x^2}} + C$

(E) $-\dfrac{2}{\sqrt{1 + x^2}} + C$

10. $\displaystyle\int_4^7 \frac{x + 3}{x^2 - 3x}\, dx =$

(A) $2 \ln 4 - 5 \ln 7$

(B) $3 \ln 4 - \ln 7$

(C) $2 \tan^{-1} 4 - \tan^{-1} 1$

(D) $5 \ln 4 - 2 \ln 7$

(E) $\ln 4 - 3 \ln 7$

11. $\displaystyle\int_1^\infty \frac{x^2\, dx}{2x^3 - 1} =$

(A) diverges

(B) $\frac{1}{2}$

(C) $\frac{1}{6}$

(D) 0

(E) $\ln\left(\frac{1}{3}\right)$

12. $\int_{-1}^{2} \frac{dx}{x^2} =$

(A) 0 (B) $-\frac{3}{2}$ (C) 3 (D) 1 (E) diverges

13. Given the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$,

(A) the Ratio Test shows that the series diverges

(B) the n–th Term Test shows that the series diverges

(C) the Integral Test shows that the series converges

(D) the Integral Test shows that the series diverges

(E) the Ratio Test shows that the series converges

14. Given the series $\sum_{n=1}^{\infty} \frac{n^2 + n + 1}{n^4 - n^2 + 1}$,

(A) the Comparison Test (limit or direct form) shows that the series converges

(B) the Comparison Test (limit or direct form) shows that the series diverges

(C) the Ratio Test shows that the series converges

(D) the Ratio Test shows that the series diverges

(E) the n–th Root Test shows that the series diverges

15. The series $\sum_{n=0}^{\infty} \frac{(-1)^{n-1}}{2n + 1}$

(A) diverges, by the Ratio Test

(B) diverges, by the Comparison Test

(C) diverges, by the Integral Test

(D) converges conditionally

(E) converges absolutely

16. The interval of convergence of the series

$$\sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^{n} n\,(x-1)^{n}$$

is

(A) $(-3,1)$ (B) $(-1,3)$

(C) $\left(-\frac{1}{2}, -\frac{3}{2}\right)$ (D) $(-2, 2)$

(E) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

17. The degree 6 term of the Maclaurin series for $\ln(1 + x^2)$ is

(A) $\frac{x^6}{6}$ (B) $\frac{x^6}{3}$ (C) $-\frac{x^6}{3}$

(D) $\frac{x^6}{6!}$ (E) $-\frac{x^6}{6!}$

18. The degree 3 term of the Maclaurin series for $\frac{\ln(1 + x)}{1 - x}$ is

(A) $-\frac{11}{6}\ x^3$

(B) $\frac{4}{3}\ x^3$

(C) $\frac{5}{6}\ x^3$

(D) $-\frac{5}{3}\ x^3$

(E) $\frac{1}{2}\ x^3$

19. The 3rd order Taylor polynomial for $f(x) = \sqrt{1 + 2x}$ at $a = 0$ is

(A) $1 + x - \frac{1}{2}\ x^2 + \frac{1}{2}\ x^3$

(B) $1 + x - \frac{1}{4}\ x^2 + \frac{1}{4}\ x^3$

(C) $1 + \frac{1}{2}\ x - \frac{1}{2}\ x^2 + \frac{1}{6}\ x^3$

(D) $1 + 2x - 2x^2 + 4x^3$

(E) $1 + 2x^2 - x^2 + 2x^3$

20. If $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(n+1)\,x^n}{n} = 2x - \frac{3x^2}{2} + \frac{4x^3}{3} - \frac{5x^4}{4} + \ldots$,

then $\int f(x)\,dx =$

(A) $x \ln(1 + x) + C$

(B) $\cos x + C$

(C) $x \sin x + C$

(D) $\frac{x^2}{1 - x} + C$

(E) $x^2 e^x + C$

21. The curve given by the parametric equations

$$x = \sqrt{t^2 + 2t + 2} \ , \qquad y = t + 1 \ , \ -\infty < t < \infty \ ,$$

most closely resembles

(A) (B) (C) (D) (E)

22. A curve given by the polar equation $r = 3 - 2r \cos \theta$. If the initial ray for the polar coordinate system is taken to be the positive x-axis in the usual way, the curve most closely resembles

(A) (B) (C) (D) (E)

23. A curve is given by parametric equations

$$x = 3t^2, \; y = 3t - t^3, \; 0 \le t \le 1.$$

The length of the curve is

(A) 4 (B) $\frac{7}{3}$ (C) $\frac{4}{3}$ (D) 10 (E) $\frac{13}{3}$

24. The polar equation $r = \sin^2\theta \sec\theta - \cos\theta$ represents a curve whose Cartesian equation is

(A) $x^3 + y = xy^2$ (B) $x^2 + y^2 = y^2 - x$
(C) $x^3 + xy^2 = y^2 - x^2$ (D) $x^3 + y^3 = y^2 - x^2$
(E) $x^2 y + y^3 = y^2$

25. The length of the curve given by the polar equation

$$r = \sqrt{1 + \cos 2\theta}\ ,\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}\ ,$$

is

(A) $\frac{\pi}{2}$ (B) 2π (C) $\frac{\pi}{\sqrt{2}}$ (D) $\sqrt{2}\ \pi$ (E) π