1. It is easy to show that the $f(x) = \ln\left(\frac{x-1}{x+1}\right)$ is defined for $x > 1$ and is an increasing function. Find its inverse, f^{-1}.

(A) $\frac{1}{\ln\left(\frac{x-1}{x+1}\right)}$ (B) $\ln\left(\frac{x+1}{x-1}\right)$ (C) $\frac{\ln x - 1}{\ln x + 1}$

(D) $e^{\frac{x-1}{x+1}}$ (E) $\frac{1+e^x}{1-e^x}$

2. $f(x) = x^3 + x - 2$ is an increasing function. $\left(f^{-1}\right)'(0) = ?$

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $-\frac{1}{2}$ (D) 3 (E) 1

3. If $y = \sqrt[3]{\dfrac{(x-1)^4}{(x+1)^2}}$, then $\left.\dfrac{dy}{dx}\right|_{x=0} = ?$

(A) -2 (B) 2 (C) -1 (D) $\frac{1}{3}$ (E) $\frac{4}{3}$

4. $\displaystyle\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\cos t}{2 + \sin t}\, dt = ?$

(A) 1 (B) $\frac{4}{3}$ (C) $\sqrt{2}$ (D) $2\ln 2$ (E) $\ln 3$

5. Which of the following curves most closely resembles the graph of $y = xe^{-x}$?

(A)

(B)

(C)

(D)

(E)

6. $2^{(\ln 3)(\log_2 e)} = ?$

(A) $\sqrt[3]{2}$ (B) e^3 (C) 3 (D) 8 (E) $\frac{\ln 3}{\ln 2}$

7. $\int_{-\ln 4}^{\ln 4} \sqrt{e^x} \; dx = ?$

(A) $\sqrt{2} - \frac{1}{\sqrt{2}}$ (B) 3 (C) $e - \frac{1}{e}$ (D) 2 (E) 1

8. A colony of bacteria grows at a rate proportional to the number present. If 500,000 are present at noon and 1,500,000 at 2 PM, then the number present at 6 PM is

(A) 13,500,000
(B) 4,500,000
(C) 40,500,000
(D) 364,500,000
(E) 3,000,000

9. $\lim_{x \to 0} \frac{\arctan x}{\sin x} = ?$

(A) 0 (B) -1 (C) $-\infty$ (D) $\frac{\pi}{2}$ (E) 1

10. $\displaystyle\int_0^{\frac{1}{2}} \frac{\text{arc sin } x}{\sqrt{1 - x^2}}\, dx = ?$

(A) $\frac{\pi}{24}$ (B) $\frac{1}{2\pi}$ (C) $\frac{\pi^2}{72}$ (D) $\frac{1}{3\sqrt{6}}$ (E) $\frac{1}{4\sqrt{2}}$

11. Where should the point P be chosen on the line segment AB so as to maximize the angle θ?

(Hint: the arc cotangent function is useful.)

(A) At a distance $\frac{\sqrt{10}}{6}$ from A

(B) At a distance $\frac{\sqrt{3}}{3}$ from A

(C) At a distance $(\sqrt{2} + \sqrt{5}) - 3$ from A

(D) At a distance $5 - 2\sqrt{5}$ from A

(E) At a distance $5 - 3\sqrt{2}$ from A