

1. $\int_0^{\infty} 2xe^{-x^2} dx = ?$

- (A) diverges (B) 1 (C) $\frac{\pi}{4}$ (D) $\ln 2$ (E) $\sqrt{2}$

2. $\int_0^2 \frac{d\theta}{\sqrt{4-\theta^2}} = ?$

- (A) 2 (B) 0 (C) $\frac{\pi}{2}$ (D) diverges (E) 1

3. Square ABCD has sides of length 1. Square EFGH is formed by connecting the midpoints of the sides of the first square, as shown in the figure below. Assume that the pattern of shaded regions in the square is continued indefinitely.

What is the total area of the shaded region?

- (A) $\frac{1}{5}$ (B) $\frac{3}{8}$
(C) $\frac{1}{4}$ (D) $\frac{1}{5}$
(E) $\frac{2}{9}$

4. Given the infinite series

1) $\sum_{n=1}^{\infty} \frac{2}{n^{7/6}}$ 2) $\sum_{n=1}^{\infty} \frac{5^n}{4^n}$ 3) $\sum_{n=1}^{\infty} \frac{n^4}{n^4 + 4}$

- (A) (1) and (2) converge, but (3) diverges
(B) all three series diverge
(C) (1) and (3) converge, but (2) diverges
(D) (1) converges, but (2) and (3) diverge
(E) (2) converges, but (1) and (3) diverge

5. Given the infinite series

$$1) \sum_{n=1}^{\infty} \frac{\cos^4 n}{n^4}$$

$$2) \sum_{n=1}^{\infty} \frac{n+2}{2n^3+n-1}$$

- (A) (1) converges, (2) diverges
- (B) (1) and (2) both converge
- (C) (1) diverges, (2) converges
- (D) (1) and (2) both diverge

6. When the Ratio Test is applied to the three infinite series

$$1) \sum_{n=1}^{\infty} \frac{n+1}{n^5+1}$$

$$2) \sum_{n=1}^{\infty} \frac{3^n}{n^3}$$

$$3) \sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$$

- (A) no information on (1), (2) diverges, (3) converges
- (B) (1) converges, (2) diverges, no information on (3)
- (C) (1) diverges, (2) converges, (3) diverges
- (D) no information on (1), (2) converges, (3) diverges
- (E) no information on (1), (2) diverges, no information on (3)

7. Which one of the following series is conditionally convergent?

(A) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^2 + 1}$

(B) $\sum (-1)^{n+1} e^{-n}$

(C) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^{3/4}}$

(D) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{7^n}{n!}$

(E) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\cos^2 n}{n^2}$

8. From the power series expansion of e^x , we can easily see that

$$1 - \frac{1}{e} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!}$$

Using the Alternating Series Error Estimate, the error involved in using only the first 6 terms is less than

(A) 0.00005

(B) 0.04

(C) 0.008

(D) .000003

(E) 0.0002

9. Which of the following has radius of convergence greater than 1?

$$\sum_{n=1}^{\infty} nx^n,$$

$$\sum_{n=0}^{\infty} 3^n x^n, \quad \sum_{n=1}^{\infty} \frac{n^2}{5^n} x^n,$$

$$\sum_{n=1}^{\infty} \frac{x^n}{(n+1)!}$$

- (A) all four
- (B) the last three
- (C) the last 2
- (D) the last 1
- (E) none

10. $\sum_{n=0}^{\infty} \frac{4^n x^n}{\sqrt{n}}$ has interval of convergence

- (A) $(-4, 4)$
- (B) $[-\frac{1}{4}, \frac{1}{4})$
- (C) $[-4, 4]$
- (D) $(-\frac{1}{4}, \frac{1}{4}]$
- (E) $(-\infty, \infty)$

11. Let $f(x) = \sum_{n=0}^{\infty} (n+1)x^n$ for $0 \leq x < 1$.

Then $\int_0^{3/4} f(x) dx =$

- (A) $\ln 3$ (B) 3 (C) 1 (D) $\frac{1}{3}$ (E) $\sqrt{2}$