1. How many of the following are true?

i) $\ln \frac{1}{3} = \int_3^1 \frac{1}{t}\, dt$

ii) $e^{-x} < 0$ for all x

iii) $2^{\pi} = e^{2 \ln \pi}$

iv) $\log_4 x = 2 \log_2 x$

(A) 0 (B) 2 (C) 3 (D) 1 (E) 4

2. The function $y = \frac{1 + 3x}{5 - 2x}$ has an inverse given by

(A) $y = \frac{5 - 2x}{1 + 3x}$ (B) $y = \frac{3 + x}{2 - 5x}$ (C) $y = \frac{5x - 1}{2x + 3}$

(D) $y = \frac{2x - 1}{3x - 5}$ (E) $y = \frac{x + 2}{5x - 3}$

3. Let $f(x) = x^3 + 2x + 1$. It is easy to see that $f(x)$ has an inverse function $g(x)$. Find $g'(1)$

(A) $\frac{1}{2}$　　(B) 1　　(C) $\frac{1}{3}$　　(D) 5　　(E) $\frac{1}{5}$

4. Let $f(x) = \frac{x^e}{e^x}$ for $x \geq 0$
The largest value f(x) attains is

(A) e　　(B) 1　　(C) $\frac{1}{e}$　　(D) π^e

(E) there is no largest value

5. The charge on a capacitor decreases at a rate proportional to the charge. If the original charge is 10 units and is 5 units 1 second later, how long does it take for the charge to decrease from 10 units to 1 unit?

(A) ln 9　　(B) $3\frac{1}{3}$　　(C) $\frac{\ln 10}{\ln 2}$

(D) $\frac{1}{2}$ ln 2　　(E) $\frac{5 \ln 2}{\ln 10}$

6. The solution of the initial value problem

$$\frac{dy}{dx} = 2xy, \quad y(-1) = 1$$

is $y =$

(A) $\sqrt{\ln x + 1}$ (B) e^{x-1} (C) $\sqrt{2(\ln x + 1)}$

(D) $e^{x^2 - 1}$ (E) $e^{\frac{1}{2}(x^2 - 1)}$

7. $\lim_{x \to 0} \csc x \bullet \ln(1 - x) = ?$

(A) 1 (B) $-\infty$ (C) 0 (D) ∞ (E) -1

8. The solution of the initial value problem

$$t\frac{dy}{dt} + 2y = t^3,\ t > 0,\ \ y(2) = 1$$

is $y =$

(A) $\frac{t^3}{5} - \frac{12}{5t^2}$ (B) $\frac{t^2}{4} + \frac{9}{4t}$ (C) $\frac{t^4}{6} - \frac{8}{3t^3}$

(D) $t + \frac{1}{2t}$ (E) $\frac{t^3}{6} - \frac{5}{2t}$

9. If $y = \arctan(\sinh x)$, then $\frac{dy}{dx} =$

(A) $\sqrt{\cosh^2 x + 1}$

(B) sech x

(C) cosh(tan x)

(D) $\dfrac{1}{\sqrt{1 - \sinh^2 x}}$

(E) $\sec^2 (\cosh x)$

10. $\displaystyle\int_0^1 x^3 e^{x^2}\,dx =$

(A) 1 (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{5}$ (E) $\frac{1}{2}$

11. $\displaystyle\int_2^3 \frac{x^2 + 1}{x^2 - 1}\,dx =$

(A) $\frac{\pi}{2}$ (B) $\pi - \frac{3}{2}$ (C) $\frac{3}{2}$ (D) $1 + \ln\frac{3}{2}$ (E) $\sqrt{\frac{3}{2}}$

12. $\displaystyle\int_3^4 \frac{1}{x^2\sqrt{25 - x^2}}\,dx =$

(A) $\frac{1}{50}$ (B) $\frac{2}{75}$ (C) $\frac{7}{300}$

(D) $\frac{3}{100}$ (E) diverges

13. $\int_1^{\infty} \frac{1}{(x+3)^{3/2}}\, dx =$

(A) 1 (B) $\frac{1}{\sqrt{2}}$ (C) diverges (D) $\frac{1}{2}$ (E) $\frac{2}{3}$

14. $\int_3^{6} \frac{1}{(x-5)^2}\, dx = ?$

(A) $\frac{1}{2}$ (B) 1 (C) $-\frac{3}{2}$ (D) diverges (E) $\frac{1}{2}\ln 5$

15. If the nth partial sum of the infinite series

$\sum_{n=1}^{\infty} a_n$ is $s_n = \frac{n}{n+1}$, then $a_n = ?$

(A) $\frac{1}{n+1}$ (B) $\frac{1}{n(n+1)}$ (C) $\frac{1}{n^2-1}$

(D) $\frac{2}{n}$ (E) $\frac{1}{n^2+n+1}$

16. How many of the following infinite series converge?

$$\sum_{n=2}^{\infty} (-1)^n \frac{\ln(n)}{\ln(n^3)}, \quad \sum_{n=1}^{\infty} \frac{3^n}{n2^n}, \quad \sum_{n=1}^{\infty} \frac{e^n}{1+2^{2n}}, \quad \sum_{n=1}^{\infty} \frac{\arctan(n)}{n^{4/3}}$$

(A) 0 (B) 1 (C) 4 (D) 3 (E) 2

17. Which of the following infinite series are absolutely convergent?

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}, \quad \sum_{n=0}^{\infty} \frac{(-1)^n}{n^{4/3}}, \quad \sum_{n=1}^{\infty} \frac{\sin n}{n^2}, \quad \sum (-1^n) \frac{2^n}{n!}$$

(A) only the last one

(B) the middle two

(C) the last three

(D) the 2^{nd} and the 4^{th}

(E) all four

18. The interval of convergence of the power series

$$\sum_{n=0}^{\infty} (-1)^n \frac{(x+2)^n}{3^n}$$ is

(A) [–3, 3) (B) (–5,1) (C) [–2, 2]

(D) (–4, 0) (E) [–6, 0)

19. The Taylor polynomial of order 4 centered at $a = 0$ for the function

$f(x) = \cos^2 x$ is

(A) $x^2 + \frac{x^4}{36}$ (B) $1 - x^2 + \frac{x^4}{12}$ (C) $x^4 \cos c$

(D) $1 - x^2 + \frac{x^4}{3}$ (E) $\frac{x}{2} - \frac{x^3}{12}$

20. The curve given parametrically by the equations

$x = \cos^2 t$, $y = \sin^2 t$

most closely resembles

(A) (B)

(C)

(D)

(E)

21. The number of points of intersection of the curves

$$r = 2 \text{ and } r = 2 \sin 2\theta$$

is

(A) 4 (B) 2 (C) 0 (D) 1 (E) 3

22. A curve is given by a polar equation

$$r = 2\sin\theta + 2\cos\theta$$

The length of the curve $\theta = 0$ to $\theta = \frac{\pi}{2}$ is

(A) 2π (B) $\frac{\pi}{\sqrt{2}}$ (C) $\frac{\pi}{2}$ (D) π (E) $\pi\sqrt{2}$

23. The area of the region that lies outside of the curve $r = 2(1 + \cos\theta)$ and inside the curve $r = 6\cos\theta$ is

(A) 12 (B) 4π (C) $2\pi + 4\sqrt{3}$ (D) $3(1 + \pi)$ (E) $\pi + 6\sqrt{2}$

24. One of the foci of the ellipse $9x^2 + 5y^2 = 45$ is at point

(A) (0,2) (B) $\left(\frac{\sqrt{14}}{3}, 0\right)$ (C) (2,0)

(D) (0,3) (E) $\left(0, \frac{\sqrt{14}}{3}\right)$

25. The polar equation

$$r = \frac{6}{2 + \cos \theta}$$

represents

(A) a hyperbola whose center has Cartesian coordinates (2, 0)

(B) a parabola whose vertex has Cartesian coordinates (2, 0)

(C) an ellipse whose center has Cartesian coordinates (–2, 0)

(D) an ellipse whose center has Cartesian coordinates (2, 0)

(E) a hyperbola whose center has Cartesian coordinates (–2, 0)