

**Math 126A: Calculus II**  
**Exam I** *September 30, 1999*

Name: \_\_\_\_\_

There are 7 problems worth of total of 80 points. You start with 20 points. To receive full credit you must show all your work and include all important steps.

1. (10 pts) Inverse Functions.

- a) Find the largest interval on the  $x$ -axis containing the number  $x = 1$  on which the function

$$f(x) = x + \frac{1}{4x}$$

is one-to-one.

- b) Find the inverse  $f^{-1}(x)$  of the function  $f(x)$  restricted to the interval determined in a).

2. (10 pts) Let  $f(x) = x^{1/x}$  for  $x > 0$ .

a) Compute  $f'(x)$ .

b) Compute  $\lim_{x \rightarrow \infty} f(x)$ .

3. (10 pts) An ice cube tray is filled with water at  $70^\circ\text{F}$  and placed in a freezer kept at  $-15^\circ\text{F}$ . Ten minutes later the water is at  $60^\circ\text{F}$ . Use Newton's Law of cooling to determine when the water will begin to freeze (reach  $32^\circ\text{F}$ ). Recall that Newton's Law of Cooling is

$$\frac{d}{dt}(T(t) - T_S) = k(T(t) - T_S)$$

where  $T(t)$  is the temperature of an object and  $T_S$  is the constant temperature of the surrounding medium.

4. (10 pts) Compute the following limits.

a)  $\lim_{x \rightarrow 0} \frac{e^x - (x + 1)}{x^2}$

b)  $\lim_{x \rightarrow \pi^-} \left( \frac{1}{\sin(x)} + \frac{1}{x - \pi} \right)$

5. (10 pts) Rates of Growth.

a) Show that  $\ln(\ln(x)) = o(\ln(x))$  as  $x \rightarrow \infty$ .

b) Show that  $100e^x + x^2$  grows at the same rate as  $e^x + x$  as  $x \rightarrow \infty$ .

6. (20 pts) Compute the following integrals.

a)  $\int_1^4 \frac{1}{\sqrt{x} + x} dx$

b)  $\int \frac{dx}{x^2 + 2x + 5}$

c)  $\int \frac{e^x dx}{\sqrt{4e^{2x} - 1}}$

d)  $\int_0^{\pi/2} x^2 \cos(x) dx$

7. (10 pts) Solve the initial value problem

$$\frac{dy}{dx} = x + xy \quad y(0) = 2$$