Math 126A: Calculus II Name:__________________________

Exam II *November 11, 1999*

There are 7 problems worth of total of 80 points. You start with 20 points. To receive full credit you must show all your work and include all important steps.

You may use a calculator.

1. (10 pts) Give the partial fraction decomposition of $\dfrac{x^2 - 2x + 2}{x^3 - 2x^2 + x}$.

2. (10 pts) Use a trigonometric substitution to integrate $\int \frac{x^2}{(x^2+1)^{5/2}}\,dx$.

3. (12 pts) Improper Integrals.

(a) Evaluate $\int_0^\infty xe^{-x^2}\,dx$

(b) Determine whether the integral $\int_1^\infty \frac{\sqrt{x^2+1}}{x^2}\,dx$ converges.

4. (12 pts) Infinite Series.

(a) Evaluate $\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right)$.

(b) Suppose $a_1 = 3$, $a_2 = 2$, and for any $N > 2$, $\sum_{n=1}^{N} a_n = \frac{N-3\sqrt{N}}{2N+\sqrt{N}}$.

Determine the value of the series $\sum_{n=3}^{\infty} a_n$.

(*Note:* The range of indices in the question is $n = 3$ to ∞.)

5. (18 pts) Determine whether the following series converge.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3 + 5n - 4}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{(2n-1)!}$

(c) $\sum_{n=2}^{\infty} \frac{1}{(\ln(n))^n}$

6. (6 pts) Determine whether the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2+1}{n^3}$ converges absolutely, converges conditionally, or diverges.

7. (12 pts)

(a) Find the interval of convergence of the power series:

$$\sum_{n=1}^{\infty} \frac{(x+1)^n}{2^n}$$

(b) Find the Taylor polynomial of order 3 for $f(x) = \ln(2 - x)$ at $x = 1$.