Math 126A: Calculus II Name:_____
Final Exam *December 16, 1999*

There are 12 problems on 12 pages worth of total of 125 points. You start with 25 points. To receive full credit you must show all your work and include all important steps.

You may use a calculator.

1. (10 pts)

   (a) Suppose $f(x)$ is a one-to-one function on the interval $[-3, 3]$ with some of the values of $f(x)$ and $f'(x)$ given in the following table.

x	$f(x)$	$f'(x)$
-2	0	$1/3$
-1	$1/2$	1
0	1	$5/3$
1	$5/2$	5
2	5	7

   Compute $(f^{-1})'(0)$, the derivative of the inverse of f at 0.

   (b) Let $f(x) = (x^2 - 4)^2$. Show that f restricted to the interval $[0, 2]$ is one-to-one and compute its inverse, $f^{-1}(x)$.

2. (10 pts) Let $f(x) = x^{\sqrt{x}}$ for $x > 0$.

(a) Compute $f'(x)$ for $x > 0$.

(b) Compute $\lim_{x \to 0^+} f(x)$.

3. (10 pts) Evaluate the following limits.

(a) $\lim_{x \to 0} \dfrac{x - \tan^{-1}(x)}{x^3}$

(b) $\lim_{x \to 0} \dfrac{1 - \cosh(x)}{x \sinh(x)}$

4. (10 pts) Water drains from a pond at a rate proportional to the depth of the water. If the depth is 4 ft at noon on Monday and 3.5 ft at noon on Tuesday, determine the depth of the water at noon on Wednesday. You may assume that the volume of water in the pond is proportional to its depth, y, so that the rate of change of y is also proportional to y.

5. (10 pts) Evaluate the following integrals.

(a) $\int_8^\infty \frac{dx}{x\sqrt{x+1}}$.

(b) $\int z \ln(z)\, dz$.

6. (15 pts) Determine whether the following series converge.

(a) $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

(b) $\sum_{n=2}^{\infty} \frac{1}{n[\ln(n)]^3}$

(c) $\sum_{n=1}^{\infty} n^2 e^{-n^2}$

7. (15 pts) Find the interval of convergence of each of the following power series.

(a) $\sum_{n=1}^{\infty} n! x^n$

(b) $\sum_{n=3}^{\infty} n^2 (x+2)^n$

(c) $\sum_{n=5}^{\infty} \frac{(x-3)^n}{n}$

8. (10 pts)

(a) Find the Taylor polynomial $P_n(x)$ for $f(x) = \sin(x)$ centered at $a = \pi/2$. (*Hint:* The polynomial should be written in terms of powers of $(x - \pi/2)$, not x.)

(b) Determine a value of n such that $P_n(x)$ approximates $f(x) = \sin(x)$ for x between $\frac{\pi}{2} - 0.1$ and $\frac{\pi}{2} + 0.1$ with an error less than 10^{-6}.

9. (5 pts) Express $\int_0^1 e^{x^2}\,dx$ as an infinite series. Be sure to include an expression for the general term of this series.

10. (10 pts) Consider the curve given by the parametric equations

$$x = t^2 - 1, \qquad y = t(t^2 - 2), \qquad -2 \leq t \leq 2$$

(a) Sketch the curve, indicating any intercepts and the direction of the curve for increasing values of t.

(b) Find the equation for the line tangent to the curve at the point $(2, \sqrt{3})$.

11. (10 pts) Determine the foci, vertices, center, asymptotes, and directrix as appropriate for the conic section given by $x^2 - y^2 - 2x + 8y - 6 = 0$.

12. (10 pts)

(a) Find the Cartesian equation for the curve given by the polar equation $r = \cos^3(\theta)$.

(b) Find the length of the curve given by the polar equation $r = \cos^2(\theta/2)$ for $0 \leq \theta \leq \dfrac{\pi}{3}$.