SIMPLE TRIGONOMETRIC FORMULAS

MATH 126: CALCULUS II

Fundamental Identities

The functions $\cos(\theta)$ and $\sin(\theta)$ are defined to be the x and y coordinates of the point at an angle of θ on the unit circle. Therefore, $\sin(-\theta) = -\sin(\theta)$, $\cos(-\theta) = \cos(\theta)$, and

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

The other trigonometric functions are defined in terms of sine and cosine:

$$\begin{aligned} \tan(\theta) &= \sin(\theta)/\cos(\theta) & \cot(\theta) &= \cos(\theta)/\sin(\theta) = 1/\tan(\theta) \\ \sec(\theta) &= 1/\cos(\theta) & \csc(\theta) &= 1/\sin(\theta) \end{aligned}$$

Dividing $\sin^2(\theta) + \cos^2(\theta) = 1$ by $\cos^2(\theta)$ or $\sin^2(\theta)$ gives

$$\begin{aligned} \tan^2(\theta) + 1 &= \sec^2(\theta) \\ 1 + \cot^2(\theta) &= \csc^2(\theta) \end{aligned}$$

Addition Formulas

The following two formulas are somewhat difficult to derive and should be memorized.

$$\begin{aligned} \sin(\alpha+\beta) &= \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta) \\ \cos(\alpha+\beta) &= \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) \end{aligned}$$

These formulas can be used to prove simple identities like $\sin(\pi/2 - \theta) = \sin(\pi/2)\cos(\theta) + \cos(\pi/2)\sin(\theta) = \cos(\theta)$, or $\cos(x - \pi) = \cos(x)\cos(\pi) - \sin(x)\sin(\pi) = -\cos(x)$. If we set $\alpha = \beta$ in the addition formulas we get the double-angle formulas:

$$\begin{aligned} \sin(2\alpha) &= 2\sin(\alpha)\cos(\alpha) \\ \cos(2\alpha) &= \cos^2(\alpha) - \sin^2(\alpha) \end{aligned}$$

The formula for $\cos(2\alpha)$ is often rewritten by replacing $\cos^2(\alpha)$ with $1 - \sin^2(\alpha)$ or replacing $\sin^2(\alpha)$ with $1 - \cos^2(\alpha)$ to get

$$\begin{aligned} \cos(2\alpha) &= 1 - 2\sin^2(\alpha) \\ \cos(2\alpha) &= 2\cos^2(\alpha) - 1 \end{aligned}$$

Solving for $\sin^2(\alpha)$ and $\cos^2(\alpha)$ yields identities important for integration:

$$\begin{aligned} \sin^2(\alpha) &= \frac{1}{2}(1 - \cos(2\alpha)) \\ \cos^2(\alpha) &= \frac{1}{2}(1 + \cos(2\alpha)) \end{aligned}$$

Derivatives and Integrals

$$\begin{aligned} \tfrac{d}{dx}\sin(x) &= \cos(x) & \tfrac{d}{dx}\sec(x) &= \sec(x)\tan(x) \\ \tfrac{d}{dx}\cos(x) &= -\sin(x) & \tfrac{d}{dx}\csc(x) &= -\csc(x)\cot(x) \\ \tfrac{d}{dx}\tan(x) &= \sec^2(x) & \tfrac{d}{dx}\cot(x) &= -\csc^2(x) \end{aligned}$$

$$\begin{aligned} \int \sin(x)\,dx &= -\cos(x) + C & \int \sec(x)\,dx &= \ln|\sec(x) + \tan(x)| + C \\ \int \cos(x)\,dx &= \sin(x) + C & \int \csc(x)\,dx &= -\ln|\csc(x) + \cot(x)| + C \\ \int \tan(x)\,dx &= \ln|\sec(x)| + C & \int \cot(x)\,dx &= -\ln|\csc(x)| + C \end{aligned}$$

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