

Math 126: Calculus II
Exam II *November 11, 1999*

Name: _____
Section: _____

There are 7 problems worth a total of 80 points on 7 pages plus a page to be used as a scratch sheet. You start with 20 points. Unless otherwise noted, each part of a problem is worth the same number of points.

Please read the questions carefully, especially the **bold** words.

You may use a calculator if you wish.

If you want partial credit on any problem, be sure to indicate **clearly** what you are doing. **Show your work** and **all** important steps. **No partial credit** will be given for any numerical answer that is not explained clearly.

1. (10 pts) Give the partial fraction decomposition of $\frac{x^2 - 2x + 2}{x^3 - 2x^2 + x}$.

Name: _____

2. (10 pts) Use a trigonometric substitution to integrate $\int \frac{x^2}{(x^2 + 1)^{5/2}} dx$.

Name: _____

3. (12 pts) Improper Integrals.

(a) Evaluate $\int_0^{\infty} x e^{-x^2} dx$

(b) Determine whether the integral $\int_1^{\infty} \frac{\sqrt{x^2 + 1}}{x^2} dx$ converges.

Name: _____

4. (12 pts) Infinite Series.

(a) Evaluate $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right)$.

(b) Suppose $a_1 = 3$, $a_2 = 2$, and for any $N > 2$,

$$\sum_{n=1}^N a_n = \frac{N - 3\sqrt{N}}{2N + \sqrt{N}}.$$

What is $\sum_{n=3}^{\infty} a_n$? (**Note:** The range of indices in the question is $n = 3$ to ∞ .)

Name: _____

5. (18 pts) Determine whether the following series converge.

(a) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3 + 5n - 4}}$.

(b) $\sum_{n=1}^{\infty} \frac{n!}{(2n - 1)!}$

Name: _____

(c) $\sum_{n=2}^{\infty} \frac{1}{(\ln(n))^n}$

6. (6 pts) Determine whether the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2 + 1}{n^3}$ converges absolutely, converges conditionally, or diverges.

Name: _____

7. (12 pts)

(a) Find the interval of convergence of the power series:

$$\sum_{n=1}^{\infty} \frac{(x+1)^n}{2^n}$$

(b) Find the Taylor polynomial of order 3 for $f(x) = \ln(2-x)$ at $x = 1$.

Name: _____