Math126, Test III
April 20, 1999

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for two hours.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 13 pages of the test.

Good Luck!

Please mark your answers with an X.

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

Multiple Choice

1.(5pts) Determine whether the following series converge or diverge.

$$1)\sum_{n=1}^{\infty}\frac{(-1)^n}{n}, \qquad 2)\sum_{n=1}^{\infty}\frac{1}{(\ln n)^n}, \qquad 3)\sum_{n=2}^{\infty}\frac{\sqrt{n^3-1}}{3n-1}.$$

(a) 1) 2) and 3) converge
(b) 1) absolutely converges, 2) and 3) diverge
(c) 1) conditionally converges, 2) and 3) diverge
(d) 1) conditionally converges, 2) absolutely converges and 3) diverge
(e) 1) 2) and 3) diverge

2.(5pts) Find the radius R of convergence of the following power series

$$\sum_{n=1}^{\infty}\frac{(x-5)^n}{n^n}.$$

(a) $R=0$ (b) $R=\infty$ (c) $R=1$ (d) $R=5$ (e) $R=\sqrt{5}$

3.(5pts) Use the definition to find the Maclaurin series for the function

$$\frac{1}{(1-2x)^2}$$

(a) $\sum_{n=1}^{\infty}(-1)^n n 2^n x^{n-1}$ (b) $\sum_{n=1}^{\infty}(-1)^n n 2^{(n+1)} x^n$ (c) $\sum_{n=1}^{\infty}(-1)^n n 2^{(n-1)} x^{n-1}$
(d) $\sum_{n=1}^{\infty}(-1)^n 2^n x^n$ (e) $\sum_{n=1}^{\infty}(-1)^n (n+1) 2^{(n-1)} x^{n-1}$

4.(5pts) Give the Maclaurin series of the function $f(x)=\sin(x^2)$.
(a) $\sum_{k=0}^{\infty}\frac{(-1)^{(2k+1)}}{(2k+1)!}x^{4k+2}$ (b) $\sum_{k=1}^{\infty}\frac{(-1)^k}{(2k+1)!}x^{4k}$ (c) $\sum_{k=0}^{\infty}\frac{(-1)^{(k+1)}}{(2k+1)!}x^{2k+2}$
(d) $\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!}x^{2k+1}$ (e) $\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!}x^{4k+2}$

5.(5pts) Give the first three nonzero terms of the Maclaurin series expansion of $e^x\sin x$.
(a) $x+x^2+\frac{2}{3}x^3$ (b) $x-x^2-\frac{1}{2}x^3$ (c) $x+3x^2+\frac{1}{6}x^3$

(d) $x + x^2 + \frac{1}{3}x^3$ (e) $x + x^2 - x^3$

6.(5pts) Which series conditionally converges?

(a) $\sum_{n=1}^{\infty} \frac{1}{n^2}$ (b) $\sum_{n=1}^{\infty} \frac{1}{n}$ (c) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ (d) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$ (e) $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$

7.(5pts) Find the sum of the following series

$$\sum_{n=0}^{\infty} e^{-n} - e^{-(n+1)}.$$

(a) diverges (b) e (c) e^{-1} (d) 1 (e) 2

8.(5pts) Determine the set of all values x such that the series

$$\sum_{n=0}^{\infty} (\ln x)^n$$

converges.

(a) $e^{-1} < x < e$ (b) diverges for all x (c) $1 < x < e$ (d) $e^{-1} \leq x < 0$ (e) converges for all x

9.(5pts) Compute

$$\lim_{x\to\infty} \frac{2(\cos(x) - 1) + 2x^2}{x^4}$$

(a) $+\infty$ (b) 0 (c) $\frac{1}{24}$ (d) 1 (e) $\frac{1}{12}$

10.(5pts) Find the third order term of the Maclaurin expansion of the solution $y(x)$ of the following initial value problem:

$$y' = y + x^2, \quad y(0) = -2$$

(a) x^3 (b) 0 (c) $\frac{x^3}{3}$ (d) $3x^3$ (e) $\frac{x^3}{3!}$

Partial Credit

11.(10pts) Find interval of convergence of the following series

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(1-x)^n}{n}.$$

12.(10pts) Find Maclaurin series representation for the function

$$\int_0^x e^{-t^2}\,dt.$$

13.(10pts) Find interval of convergence of the following power series:

$$\sum_{n=1}^{\infty} \frac{(2n)^2}{3}(x-2)^n.$$

Be sure to investigate the endpoints of the interval.

14.(10pts) If $0 < x < 0.5$, use Alternating Series Theorem and the Binomial Theorem to show that $\sqrt{1+x} \approx 1 + 0.5x$ with an error less than 0.032. Notice that the series is alternating after the first term.

15.(10pts) Find 4 first terms of the Taylor series for $f(x) = \cos x$ about $a = \pi$.