1. One of the following is <u>false</u> (as $x \to \infty$)

(A) $e^{2x} = o(e^{3x})$ (B) $x^2 = o(x \ln x)$ (C) $x = o(x^2 - \ln x)$

(D) $x^3 = o(e^x)$ (E) $\ln x = o(\sqrt{x})$

2. The region under the curve
$y = \sqrt{x}\, e^x,\ 0 \le x \le 1,$ is
rotated about the x-axis. The volume of the resulting solid is

(A) $\frac{\pi}{4}(3e^2 - 1)$ (B) $\pi(e - 1)$ (C) $\frac{\pi}{4}(e^2 - 1)$

(D) $\frac{\pi}{4}(e^2 + 1)$ (E) π

3. $\displaystyle\int_0^{\pi} x \sin x \, dx =$

(A) $\frac{2\pi}{3}$ (B) $\frac{1}{4}$ (C) π^2 (D) π (E) -2

4. $\displaystyle\int_2^{4} \frac{3x - 1}{x^3 - x}\, dx =$

(A) $\ln 2 + 3 \ln 3 - 2 \ln 5$ (B) $\ln 2 + 2 \ln 3 - \ln 5$
(C) $-\ln 2 + 3 \ln 3 - \ln 5$ (D) $2 \ln 2 - \ln 3 + \ln 5$
(E) $2 \ln 2 - 2 \ln 3 + \ln 5$

5. $\displaystyle\int_2^{4} \frac{x^3 - x^2 + 2x - 1}{x^2 - x}\, dx =$

(A) $3 + 4 \ln\left(\frac{2}{3}\right)$ (B) $6 + \ln 6$ (C) $4 + \ln 6$

(D) $\ln 6$ (E) $4 + 3 \ln\left(\frac{3}{2}\right)$

6. To find the integral $\int \sqrt{3 + 2x - x^2}\, dx$, the method of trigonometric substitution can be used. A suitable substitution, and the resulting trigonometric integral, are

(A) $x = 2 \sin \theta + 1; \int 4 \cos^2 \theta \, d\theta$

(B) $x = 2 \sin \theta + 1; \int 2 \cos \theta \, d\theta$

(C) $x = 2 \sin \theta - 1; \int 4 \cos^2 \theta \, d\theta$

(D) $x = 2 \sin \theta - 1; \int 2 \cos \theta \, d\theta$

(E) $x = 2 \tan \theta + 1; \int 4 \sec^3 \theta \, d\theta$

7. $$\int_0^2 \frac{dx}{(x-1)^2} =$$

(A) $\frac{2}{3}$ (B) diverges (C) -2 (D) 0 (E) 2

8. $$\int_1^\infty \frac{dx}{x^2 + x} =$$

(A) diverges (B) $\ln 2$ (C) $\frac{\pi}{4}$ (D) $\frac{1}{2} \ln 2$ (E) $\frac{1}{2}$

9. A sequence $\{a_n\}$ is given, with $a_1 = 1$, $a_2 = 1$, and with the recursion formula

$$a_{n+2} = a_{n+1} + \left((-1)^{a_n}\right)a_n.$$

Then, $a_{11} =$

(A) 1 B) 7 (C) 0 (D) –1 (E) 8

10. $\lim_{n\to\infty} \frac{(-1)^n \sin n}{n} =$

(A) ∞ (B) 1 (C) does not exist (D) π (E) 0

11. $\lim_{n\to\infty} \frac{\ln(n^2+n)}{\ln(n+1)} =$

(A) does not exist (B) ∞ (C) 0 (D) ln 2 (E) 2

12. The partial fraction expression for $\frac{2x+1}{x^2(x+1)^2}$ is

$$\frac{2x+1}{x^2(x+1)^2} = \frac{1}{x^2} - \frac{1}{(x+1)^2}.$$

From this, one can deduce that

$$\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2} =$$

(A) diverges (B) 0 (C) $-\frac{1}{4}$ (D) $\frac{3}{4}$ (E) 1

13. Let $\sum_{n=1}^{\infty} a_n$ be an infinite series, with partial sums $s_1, s_2, s_3, \ldots$.

Which of the following statements is true?

(A) If $\sum_{n=1}^{\infty} a_n$ diverges, then $\lim_{n\to\infty} a_n \neq 0$.

(B) If $a_n \geq 0$ for all n, then the series converges.

(C) If $\lim_{k\to\infty} s_k = 0$, then $\lim_{n\to\infty} a_n = 0$.

(D) If $\lim_{n\to\infty} a_n = 0$, then $\lim_{k\to\infty} s_k = 0$.

(E) If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{k\to\infty} s_k = 0$.

14. $\sum_{n=0}^{k} 2^n =$

(A) $2^k + 2$ (B) $2^k + 1$ (C) $2^{k+1} + 1$ (D) $2^k - 1$ (E) $2^{k+1} - 1$

15. $\sum_{n=0}^{\infty} \left(\frac{3^{n-1}}{4^n} \right) =$

(A) $\frac{8}{5}$ (B) $\frac{10}{3}$ (C) diverges (D) $\frac{4}{3}$ (E) $\frac{5}{6}$

16. Given the infinite series

(1) $\sum_{n=1}^{\infty} \left(\frac{3}{2} \right)^{n-1}$ (2) $\sum_{n=1}^{\infty} \frac{n^2}{n^2 + 1}$ (3) $\sum_{n=1}^{\infty} \left(\frac{2}{3} \right)^{n-1}$

(A) (1) converges, (2) diverges, (3) diverges

(B) (1) diverges, (2) converges, (3) converges

(C) (1) diverges, (2) diverges, (3) converges

(D) (1) converges, (2) diverges, (3) converges

(E) (1) diverges, (2) converges, (3) diverges