1. Which is a correct statement of the Comparison Test (direct or limit form) for convergence of series?

Given two positive term series $\sum a_n$, $\sum b_n$,

(A) if $a_n \geq b_n$ for all values of n, and $\sum b_n$ converges, then $\sum a_n$ converges

(B) if $\lim\limits_{n\to\infty} \frac{a_n}{b_n} = 0$, and $\sum b_n$ diverges, then $\sum a_n$ diverges

(C) if $\lim\limits_{n\to\infty} \frac{a_n}{b_n} = \infty$, and $\sum b_n$ converges, then $\sum a_n$ converges

(D) if $a_n \leq b_n$ for all n, and $\sum b_n$ diverges, then $\sum a_n$ diverges

(E) if $a_n \geq b_n$ for all n, and $\sum b_n$ diverges, then $\sum a_n$ diverges

2. Given the infinite series

(1) $\sum\limits_{n=1}^{\infty} \frac{n+1}{n^3+n^2+2}$ (2) $\sum\limits_{n=1}^{\infty} \frac{2+(-1)^n}{n}$ (3) $\sum\limits_{n=1}^{\infty} \sqrt{\frac{n+1}{n^3+n^2+2}}$

(A) All three series converge (B) All three series diverge

(C) (1) converges, (2) diverges, (3) diverges

(D) (1) converges, (2) converges, (3) diverges

(E) (1) diverges, (2) converges, (3) converges

3. When the Ratio Test is applied to the two infinite series

(1) $\sum\limits_{n=1}^{\infty} n^{-2}\, 2^{-n}$

(2) $\sum\limits_{n=1}^{\infty} \frac{n!}{(2n+1)!}$

the information it provides is

(A) (1) converges, no information on (2)

(B) (2) diverges, no information on (1)

(C) (1) converges, (2) diverges (D) (1) and (2) both diverge

(E) (1) and (2) both converge

4. Which one of the folowing series is absolutely convergent?

(A) $\sum_{n=1}^{\infty} \frac{2 + \cos n}{n}$ (B) $\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$

(C) $\sum_{n=1}^{\infty} \frac{2n + (-1)^n}{n^2}$ (D) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n + \sqrt{n}}$

(E) $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{2n^2 + n}$

5. Given the infinite series

(1) $\sum_{n=1}^{\infty} \frac{n}{2^n}$ (2) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

(A) The n–th Root Test shows that (1) diverges, and the Integral Test shows that (2) diverges

(B) The Ratio Test shows that (1) diverges, and the Ratio Test shows that (2) converges

(C) The Ratio Test shows that (1) converges, and the Integral Test shows that (2) converges

(D) The Comparison Test shows that (1) converges, and the Ratio Test shows that (2) diverges

(E) The n–th Root Test shows that (1) converges, and the Comparison Test shows that (2) diverges

6. Which one of the following statements is true?

(A) If $\sum a_n$ is an alternating series which converges, then $\sum a_n$ converges absolutely

(B) If $\sum a_n$ converges conditionally, then $\sum |a_n|$ converges absolutely

(C) If $\sum a_n$ is an alternating series, and $a_n \to 0$ as $n \to \infty$, then $\sum a_n$ converges absolutely

(D) If $\sum |a_n|$ diverges, then $\sum a_n$ diverges

(E) If $\sum a_n$ converges conditionally, then $\sum |a_n|$ diverges

7. One of the endpoints of the interval of convergence of the series

$$\sum_{n=0}^{\infty} 2^n n^2 (x-1)^n$$

is

(A) 3 (B) $\frac{2}{3}$ (C) $\frac{1}{2}$ (D) 2 (E) $\frac{1}{3}$

8. The degree 6 term of the Maclaurin series for $\sin(x^2)$ is

(A) $\frac{1}{6} x^6$ (B) $\frac{1}{3} x^6$ (C) $-\frac{1}{6!} x^6$ (D) $-\frac{1}{6} x^6$ (E) $\frac{1}{6!} x^6$

9. The third order Taylor polynomial for $f(x) = e^{-x} \cos x$ is

(A) $1 - x + \frac{2}{3} x^3$ (B) $1 + x - \frac{1}{3} x^3$ (C) $1 + x - \frac{2}{3} x^3$

(D) $1 - x + \frac{1}{2} x^2 - \frac{1}{3} x^3$ (E) $1 - x + \frac{1}{3} x^3$

10. A function $f(x)$ is given by a series

$$f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n\ 3^{n-1}}, \qquad -3 < x < 3.$$

The derivative of $f(x)$ at $x = 1$ is $f'(1) =$

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) $\frac{3}{4}$ (D) $\ln\left(\frac{2}{3}\right)$

(E) $\ln\left(\frac{3}{2}\right)$

11. The approximate value of $\ln(1.4)$ obtained by using the second order Taylor polynomial for $f(x) = \ln x$ at $a = 1$ is

(A) 0.48 (B) 0.18 (C) 0.42 (D) 0.32 (E) 0.24

12. According to Taylor's theorem, the size of the error in the approximation to $\ln(1.4)$ referred to in question 11 is equal to

(A) $\frac{1}{6} c^3$, where $0 < c < 0.4$ (B) $\frac{1}{6} \ln c$, where $1 < c < 1.4$

(C) $\frac{0.072}{c^3}$, where $1 < c < 1.4$ (D) $\frac{0.064}{3\ c^3}$, where $1 < c < 1.4$

(E) $(0.42)\ln c$, where $1 < c < 1.4$

13. The coefficient of x^3 in the Maclaurin series for $\dfrac{1}{\sqrt{1 + x}}$ is

(A) $-\dfrac{1}{2}$ (B) $\dfrac{3}{32}$ (C) $\dfrac{1}{16}$ (D) $-\dfrac{5}{16}$ (E) $\dfrac{1}{8}$

14. $\displaystyle\int_0^1 x^2 \cos(x^2)\, dx =$

(A) $\dfrac{1}{3} - \dfrac{1}{7(2!)} + \dfrac{1}{11(4!)} - \dfrac{1}{15(6!)} + \cdots$

(B) $\dfrac{1}{3} - \dfrac{1}{3\cdot 5(2!)} + \dfrac{1}{3\cdot 9(4!)} - \dfrac{1}{3\cdot 13(6!)} + \cdots$

(C) $\dfrac{1}{5} - \dfrac{1}{9(3!)} + \dfrac{1}{13(5!)} - \dfrac{1}{17(7!)} + \cdots$

(D) $\dfrac{1}{3\cdot 3} - \dfrac{1}{3\cdot 7(3!)} + \dfrac{1}{3\cdot 11(5!)} - \dfrac{1}{3\cdot 15(7!)} + \cdots$

(E) $1 - \dfrac{1}{4(2!)} + \dfrac{1}{8(4!)} - \dfrac{1}{12(6!)} + \cdots$

15. The equation $y^2 = 20 - 2x^2$ represents

(A) a hyperbola with a focus at $\left(\sqrt{30},\ 0\right)$

(B) a hyperbola with a focus at $\left(0,\ \sqrt{10}\right)$

(C) an ellipse with a focus at $\left(0,\ \sqrt{10}\right)$

(D) an ellipse with a focus at $\left(\sqrt{10},\ 0\right)$

(E) a hyperbola with a focus at $\left(0,\ \sqrt{30}\right)$

16. A line segment PQ of length 3 moves in such a way that Q always lies on the x-axis, and the intersection of PQ with the y–axis is the point R at distance 1 from Q. The point P traces out a curve whose parametric equations in terms of the angle t shown in the diagram are

(A) $x = 2\cos t,\ y = 3\sin t$ (B) $x = 3\cos t - 1,\ y = 2\sin t + 1$

(C) $x = 3\cos t,\ y = 4\sin t$ (D) $x = \dfrac{2}{3}\cos t,\ y = \sin t$

(E) $x = \frac{3}{4}\cos t, \quad y = \sin t$

17. The curve given by the parametric equations

$$x = \sqrt{1 - t^4}\,, \qquad y = t^2\,, \qquad -1 \le t \le 0,$$

most closely resembles

NO GRAPHS