Math 126: Calculus II Name:_____
Final *December 15, 2000* Instructor:_____

There are 14 problems on 10 pages worth a total of 120 points. You start with 30 points. Each part of a problem is worth the same number of points.

You may use a calculator if you wish.

To receive partial credit on a problem, you must *show your work and all important steps.* No credit will be given for an answer if no work is shown.

1. (12 pts) Consider the following information about functions f and g.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	-3	-3	0
1	0	-4	0	5
2	-1	-7	8	6

a) Suppose in addition f is one-to-one on the interval $[-1, 3]$. Compute $(f^{-1})'(0)$, the derivative of the inverse of f at 0.

b) Calculate the derivative of $\sinh(2f(x) + g(x))$ at $x = 0$.

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2. (6 pts) Compute $\frac{d}{dx} x^{\tan^{-1} x}$.

3. (6 pts) Einsteinium-253 decays at a rate proportional to the amount present. Determine the half-life if this material has 2/3 of its original mass left after 11.7 days.

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4. (12 pts) Evaluate the following limits.

a) $\lim_{x \to 0} \dfrac{\sin(x^{10}) - x^{10}}{x^{30}}$

b) $\lim_{x \to \infty} x^2 e^{-x}$

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5. (24 pts) Evaluate the integrals.

a) $\int \frac{e^x}{\sqrt{1-4e^{2x}}}\,dx$

b) $\int x^2 \ln(x)\,dx$

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c) $\int \frac{1}{x^3 - x^2}\, dx$

d) $\int \frac{\sqrt{4 - x^2}}{x^2}\, dx$

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6. (12 pts) Determine whether the following series converge:

a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\ln n + n}$

b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

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7. (6 pts) Find the sum of the series $\sum_{n=1}^{\infty}(-1)^n e^{-n}$.

8. (6 pts) Find the Taylor polynomial of order 4, $P_4(x)$, generated by $\sin(x)$ centered at $\pi/2$.

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9. (6 pts) Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{2^n}{n}(x-1)^n$.

10. (6 pts) Estimate the error of approximating e^{-x} by $1 - x + \frac{1}{2}x^2$ for $|x| < 1/2$.

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11. (6 pts) Find the equation of the line tangent to the curve parameterized by

$$\begin{aligned} x &= t + \cos(\pi t) \\ y &= t - \sin(\pi t) \end{aligned}$$

at the point $t = 2$.

12. (6 pts) Find the length of the curve parameterized by

$$\begin{aligned} x &= t^3 + 1 \\ y &= t^2 - 1 \end{aligned}$$

for $0 \leq t \leq 1$.

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13. (6 pts) Find the area inside both of the curves $r = \sin(\theta)$ and $r = 1 + \cos(\theta)$.

14. (6 pts) Use power series to evaluate the integral $\int_0^x e^{t^3}\, dt$.