

1.(6pts) Which equation below is the equation of an ellipse with its major axis the x -axis?

(a) $\frac{x^2}{25} + \frac{y^2}{36} = 1$ (b) $\frac{x^2}{36} + \frac{y^2}{25} = 1$ (c) $\frac{x^2}{25} - \frac{y^2}{36} = 1$ (d) $\frac{x^2}{36} - \frac{y^2}{25} = 1$ (e) $\frac{x^2}{36} + \frac{y^2}{25} = 0$

2.(6pts) Which equation below is that of a hyperbola with foci $(\pm 4, 0)$?

(a) $\frac{x^2}{5} - \frac{y^2}{3} = 1$ (b) $-\frac{x^2}{5} + \frac{y^2}{3} = 1$ (c) $\frac{x^2}{14} - \frac{y^2}{2} = 1$ (d) $\frac{x^2}{9} - \frac{y^2}{7} = 0$ (e) $-\frac{x^2}{14} + \frac{y^2}{2} = 1$

3.(6pts) The graph of $x = \frac{y^2}{20}$ is a parabola with directrix the line

- (a) $x = 5$ (b) $y = 5$ (c) $x = 0$ (d) $x = -5$ (e) $y = -5$

4.(6pts) Find the slope of the tangent line to the parameterized curve $x = t^2 + 3t + 1$, $y = t^3 - 2t$ when $t = 2$.

- (a) $\frac{10}{7}$ (b) $\frac{7}{10}$ (c) 1 (d) $\frac{-2}{7}$ (e) $\frac{-7}{2}$

5.(6pts) Which integral below represents the arclength of the cycloid $x = a(t - \sin t)$, $y = a(1 - \cos t)$; $0 \leq t \leq 2\pi$?

(a) $\sqrt{2a^2} \int_0^{2\pi} \sqrt{1 - \cos t + \sin t} dt$

(b) $\sqrt{2a^2} \int_0^{2\pi} \sqrt{t - \sin t + 1 - \cos t} dt$

(c) $\sqrt{2a^2} \int_0^{2\pi} \sqrt{1 - \cos t} dt$

(d) $\sqrt{2a^2} \int_0^{2\pi} \sqrt{1 + \cos t} dt$

(e) $\sqrt{2a^2} \int_0^{2\pi} \sqrt{1 - \sin t} dt$

6.(6pts) The parameterized curve $x = t^3 + 2t$, $y = \cos t$; $-\infty < t < \infty$ is also the graph of a function $y = f(x)$. What is the coefficient of x^2 in the Mclaurin series expansion for $f(x)$?

(a) $\frac{-1}{2}$

(b) $\frac{-1}{8}$

(c) 0

(d) $\frac{1}{8}$

(e) $\frac{1}{2}$

7.(6pts) The function $f(x) = x \sin x$ has one critical point for $-\frac{\pi}{2} < x < \frac{\pi}{2}$. Where is it and determine whether it is a local minima, maxima or neither.

- (a) $x = 0$, local max (b) $x = 0$, local min (c) $x = 0$ neither
(d) $x = \frac{\pi}{4}$, local max (e) $x = \frac{\pi}{4}$, local min

8.(6pts) Which number below is equal to $\log_3(81)$?

- (a) -4 (b) -2 (c) 0 (d) 2 (e) 4

9.(6pts) Let $f(x) = x + \ln x$ for $x > 0$. Find $\frac{df^{-1}}{dx}(e + 1)$.

- (a) f is not one to one (b) e (c) $\frac{1}{e+1}$ (d) $\frac{e}{e+1}$ (e) $1 + \frac{1}{e}$

10.(6pts) A certain bacteria culture, undergoing natural growth, doubles in size after 4 minutes. If there were 100 specimens at time $t = 0$, when will the number have increased to 1600 specimens?

- (a) 2 weeks (b) 1 day (c) 3 hours, 20 minutes (d) 2 hours (e) 16 minutes

11.(6pts) Let $f(x) = \int_0^x e^{-t^2} dt$. Find the Mclaurin series for $f(x)$.

- (a) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{(n+1)!}$ (b) $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{n!}$ (c) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)n!}$
(d) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{3n}}{(3n)n!}$ (e) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n)!}$

12.(6pts) Calculate $\lim_{u \rightarrow \infty} \frac{u^3 + 5u^2 - 2u + 10}{3u^2 + 7u - 8}$.

- (a) ∞ (b) $\frac{5}{3}$ (c) $\frac{-2}{7}$ (d) $\frac{-5}{4}$ (e) $\frac{1}{3}$

13.(6pts) The solution to the initial value problem $xy' = y + x^3$, $y(1) = 1$ is

(a) $y = x^3$ (b) $y = e^{x-1} + 1$ (c) $y^2 + y = x^2 + x$ (d) $y = \frac{x^2 + 2}{3}$ (e) $y = \frac{x^3}{4} + \frac{3}{4x}$

14.(6pts) The improper integral $\int_1^{\infty} \frac{1}{x^{1.01}} dx$

- (a) converges to $\frac{1}{1.01}$ (b) converges to 1.01 (c) converges to .01
(d) converges to 100 (e) diverges

15.(6pts) The partial fraction expansion of $\frac{x+7}{x^2+4x+3}$ is

(a) $\frac{2}{x-1} + \frac{3}{x-3}$

(b) $\frac{3}{x+1} - \frac{2}{x+3}$

(c) $\frac{5}{x+1} + \frac{4}{x+3}$

(d) $\frac{4}{x+1} - \frac{3}{x+3}$

(e) $\frac{x+7}{x^2+4x+3}$

16.(6pts) $\int_0^3 xe^x dx$

(a) $3e^3$

(b) $3e^3 - 3$

(c) $2e^3 + 1$

(d) $2e^3$

(e) $3e^2$

19.(6pts) Determine whether the following series converge or diverge.

$$1) \sum_{n=1}^{\infty} \frac{(-1)^n}{n}, \quad 2) \sum_{n=1}^{\infty} \frac{1}{n!}, \quad 3) \sum_{n=2}^{\infty} \frac{\sqrt[3]{n^3 - 1}}{3n - 1}.$$

- (a) 1) 2) and 3) converge
- (b) 1) absolutely converges, 2) and 3) diverge
- (c) 1) conditionally converges, 2) and 3) diverge
- (d) 1) conditionally converges, 2) absolutely converges and 3) diverge
- (e) 1) 2) and 3) diverge

20.(6pts) Find the radius R of convergence of the following power series

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} (x - 5)^n .$$

- (a) $R = 0$
- (b) $R = \infty$
- (c) $R = 4$
- (d) $R = 5$
- (e) $R = \sqrt{5}$

21.(6pts) All of the series below have radius of convergence 1. Which one conditionally converges at both endpoints of its interval of convergence?

(a) $\sum_{n=1}^{\infty} (-1)^n \frac{x^{n-1}}{n}$

(b) $\sum_{n=1}^{\infty} (-1)^n \frac{x^{2n-1}}{n}$

(c) $\sum_{n=1}^{\infty} (-1)^n \frac{x^{2n-1}}{n^2}$

(d) $\sum_{n=1}^{\infty} (-1)^n \frac{x^{3n}}{n}$

(e) $\sum_{n=1}^{\infty} (-1)^n \frac{x^{3n}}{n^3}$

22.(6pts) What is the behavior of the series $\sum_{n=1}^{\infty} \frac{1}{(\arctan n)^2(1+n^2)}$?

(a) It converges absolutely. (b) It converges conditionally. (c) It diverges.

23.(6pts) Give the first three nonzero terms of the Maclaurin series expansion of $e^{x^2} \sin x$.

(a) $x - \frac{1}{3}x^3 - \frac{1}{120}x^5$ (b) $1 + 3x^2 + \frac{1}{6}x^4$ (c) $x - x^3 + x^7$
(d) $x + \frac{5}{6}x^3 + \frac{41}{120}x^5$ (e) $x + x^2 - x^3$

24.(6pts) Which series conditionally converges?

(a) $\sum_{n=2}^{\infty} \frac{1}{n^2 \ln n}$ (b) $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ (c) $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n \ln n}$ (d) $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n^2 \ln n}$ (e) $\sum_{n=2}^{\infty} \frac{1}{n^2 + n \ln n}$

25.(5pts) Find the sum of the following series

$$\sum_{n=0}^{\infty} \frac{2^{n-2}}{3^n} .$$

(a) diverges

(b) $\frac{3}{4}$

(c) $e^{2/3}$

(d) 1

(e) 3

Name: _____

Instructor-section: Bullwinkle

Math126, Final

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- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for two hours.
- You will only hand in this page - the rest of the test is yours to keep.
- Be sure that you have all 14 pages of the test.

Good Luck!

Please mark your answers with an X.

1.	(a)	(●)	(c)	(d)	(e)	13.	(a)	(b)	(c)	(d)	(●)
2.	(a)	(b)	(●)	(d)	(e)	14.	(a)	(b)	(c)	(●)	(e)
3.	(a)	(b)	(c)	(●)	(e)	15.	(a)	(●)	(c)	(d)	(e)
4.	(●)	(b)	(c)	(d)	(e)	16.	(a)	(b)	(●)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(●)	17.	(a)	(b)	(●)	(d)	(e)
6.	(a)	(●)	(c)	(d)	(e)	18.	(●)	(b)	(c)	(d)	(e)
7.	(a)	(●)	(c)	(d)	(e)	19.	(a)	(b)	(c)	(●)	(e)
8.	(a)	(b)	(c)	(d)	(●)	20.	(a)	(b)	(●)	(d)	(e)
9.	(a)	(b)	(c)	(●)	(e)	21.	(a)	(●)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(●)	22.	(●)	(b)	(c)		
11.	(a)	(b)	(●)	(d)	(e)	23.	(a)	(b)	(c)	(●)	(e)
12.	(●)	(b)	(c)	(d)	(e)	24.	(a)	(b)	(●)	(d)	(e)
						25.	(a)	(●)	(c)	(d)	(e)