Name:		
Instructor:	Bullwinkle	
Math 126		
Exam III		
April 24, 2001		

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for one hour.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 9 pages of the test.

## Good Luck!

PLE	EASE MA	RK YOUR A	NSWERS V	VITH AN X	, not a circle!
1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)

DO NOT WRITE IN THIS BOX!		
Total multiple choice:		
9.		
10.		
11.		
12.		
Total:		

## Multiple Choice

1.(6 pts.) What can be said about the improper integral

$$\int_{1}^{\infty} xe^{-x} dx?$$

- (a) It converges to  $e^{-1}$ . (b) It diverges. (c) It converges to  $\pi$ .
- (d) It converges to 1. (e) It converges to  $2e^{-1}$ .
- **2.**(6 pts.) The sequence given by  $a_n = (\frac{n}{n+1})^n$
- (a) converges to  $\pi$ . (b) converges to 1. (c) diverges.
- (d) converges to  $e^{-1}$ . (e) converges to  $e^2$ .
- **3.**(6 pts.) The series  $\sum_{n=1}^{\infty} \frac{2}{5^{n+2}}$
- (a) converges to  $\frac{1}{50}$ . (b) converges to  $\pi$ . (c) diverges.
- (d) converges to 2. (e) converges to  $\frac{1}{10}$ .
- **4.**(6 pts.) The sequence  $a_n = \frac{2^n}{(n+1)!}$  for  $n \ge 1$  is
- (a) nondecreasing and convergent.
- (b) nonincreasing and convergent.
- (c) nonincreasing and divergent.
- (d) neither nonincreasing nor nondecreasing.
- (e) nondecreasing and divergent.

**5.**(6 pts.) The series  $\sum_{n=2}^{\infty} \frac{1}{\ln n}$ 

- (a) converges to 1. (b) diverges. (c) converges to  $\pi$ .
- (d) converges to 25. (e) converges to e.

**6.**(6 pts.) Let  $\sum_{n=1}^{\infty} a_n$  and  $\sum_{n=1}^{\infty} b_n$  be two series with nonnegative terms. Which of the following is always a correct statement?

- (a) If  $\sum_{n=1}^{\infty} a_n$  diverges and  $\sum_{n=1}^{\infty} b_n$  diverges then  $\sum_{n=1}^{\infty} (a_n b_n)$  converges.
- (b) If  $\sum_{n=1}^{\infty} a_n$  converges and  $\sum_{n=1}^{\infty} b_n$  converges then  $\sum_{n=1}^{\infty} \frac{a_n}{b_n}$  converges provided all  $b_n \neq 0$ .
- (c) If  $\sum_{n=1}^{\infty} a_n$  converges and  $\sum_{n=1}^{\infty} b_n$  diverges then  $\sum_{n=1}^{\infty} (a_n + b_n)$  diverges.
- (d) If  $\sum_{n=1}^{\infty} a_n$  diverges and  $\sum_{n=1}^{\infty} b_n$  diverges then  $\sum_{n=1}^{\infty} \frac{a_n}{b_n}$  diverges.
- (e) None of the above.

7.(6 pts.) For which values of x does the power series

$$\sum_{n=1}^{\infty} (\ln n) \left(\frac{x}{2}\right)^n$$

converge?

- (a) -1 < x < 1. (b) all values of x. (c) x = 0 only. (d) -2 < x < 2.
- (e)  $\frac{-1}{\ln 2} < x < \frac{1}{\ln 2}$ .

8.(6 pts.) The series

$$\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$$

- (a) diverges because  $\lim_{n\to\infty} \frac{(-1)^{n+1}}{\sqrt{n}} \neq 0$ .
- (b) converges absolutely.
- (c) diverges because the terms alternate.
- (d) diverges even though  $\lim_{n\to\infty} \frac{(-1)^{n+1}}{\sqrt{n}} = 0$ .
- (e) does not converge absolutely but does converge conditionally.

## Partial Credit

9.(13 pts.) Does the series

$$\sum_{n=1}^{\infty} \frac{(n!)^n}{(n^{2n})}$$

converge or diverge? Show your reasoning and state clearly any theorems or tests you are using.

10.(13 pts.) Does the integral

$$\int_{1}^{\infty} \frac{dx}{\sqrt{x}(1+x)}$$

converge or diverge? Show all of your work and state clearly and precisely any theorems you are using.

**11.**(13 pts.) Find the **interval** of convergence of the series  $\sum_{n=1}^{\infty} \frac{x^{2n}}{2^n n^2}$ . Be sure to check the end points.

4

**12.**(13 pts.)

(a) Show that

$$\sum_{n=0}^{\infty} (-1)^n x^{2n} = \frac{1}{1+x^2}$$

provided that |x| < 1.

(b) Find

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)(\sqrt{3})^{2n+1}}.$$

(Hint: First use term-by-term integration on the series in part (a).)

Name:	ANSWERS	
Instructor:	ANSWERS	
Math 126		
Exam III		
April 24, 2001	_	

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for one hour.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 9 pages of the test.

## Good Luck!

PLF	EASE MA	RK YOUR A	ANSWERS V	VITH AN X	, not a circle!
1.	(a)	(b)	(c)	(d)	(●)
2.	(a)	(b)	(c)	(ullet)	(e)
3.	(ullet)	(b)	(c)	(d)	(e)
4.	(a)	(ullet)	(c)	(d)	(e)
5.	(a)	(ullet)	(c)	(d)	(e)
6.	(a)	(b)	(ullet)	(d)	(e)
7.	(a)	(b)	(c)	(ullet)	(e)
8.	(a)	(b)	(c)	(d)	(•)

DO NOT WRITE IN THIS BOX!		
Total multiple choice:		
9.		
10.		
11.		
12.		
Total:		