Name: ______________________________

Instructor: ______________________________

Math 126, Final

May 7, 2001

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- Be sure that you have all 14 pages of the test.
- No calculators are to be used.
- The exam lasts for two hours.
- You are to hand in just the front page.

Good Luck!

Please mark your answers with an **X**! Do NOT circle them!

The dotted lines in the answer box indicate page breaks.

1.	(a)	(b)	(c)	(d)	(e)	15.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)	16.	(a)	(b)	(c)	(d)	(e)
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11.	(a)	(b)	(c)	(d)	(e)	25.	(a)	(b)	(c)	(d)	(e)
12.	(a)	(b)	(c)	(d)	(e)	Score 1: ____			Score 4: ____		
13.	(a)	(b)	(c)	(d)	(e)	Score 2: ____					
14.	(a)	(b)	(c)	(d)	(e)	Score 3: ____			Total: ____		

1.(6 pts.) Let $f(x) = 2x^3 + 1$ and let f^{-1} denote the inverse function. Then $(f^{-1})'(17) =$

(a) $\frac{1}{24}$ (b) $\frac{1}{3}$ (c) $\frac{1}{7}$ (d) $\frac{24}{17}$ (e) $\frac{1}{17}$

2.(6 pts.) $\int_e^{e^2} \frac{1}{x(\ln x)^2}\,dx =$

(a) 2 (b) $2e^2 - e$ (c) $\frac{1}{2}$ (d) Diverges (e) $\frac{1}{2e^2} - \frac{1}{e}$

3.(6 pts.) If $y'(t) = t \cdot \cos(t^2)$ and $y(0) = 1$, then $y\left(\sqrt{\frac{\pi}{2}}\,\right) =$

(a) $\frac{3}{2}$ (b) $\sqrt{\frac{\pi}{2}}$ (c) -2 (d) $\sqrt{\frac{\pi}{2}} - 1$ (e) $\sqrt{\frac{\pi}{2}} \cdot \frac{3}{2}$

4.(6 pts.) The solution to the initial value problem

$$x\frac{dy}{dx} + x^2y + x^2 = 0 \qquad y(0) = 0$$

is

(a) $y = e^{-\frac{x^2}{2}} - 1$ (b) $y = 1 - e^{-x}$

(c) $y = xe^x$ (d) $y = e - e^{-\frac{x^2}{2}+1}$

(e) $y = e^{-x} - 1$

5.(6 pts.) Assuming uniform density δ, the moment about the y–axis of the plane region bounded by the axes and the line $y = 6 - 3x$ is

(a) $3\frac{1}{3}\delta$ (b) 3δ (c) 3.5δ (d) 4δ (e) 4.5δ

6.(6 pts.) The solution to the initial value problem

$$y' = \frac{\sin x}{2y+1} \qquad y(0) = 2$$

satisfies the implicit equation

(a) $2y + 1 = 6 - e^{-\cos x}$ (b) $y^2 + y = 7 - \cos x$ (c) $2y + 1 = 5e^{-\cos x}$

(d) $y^2 + y = 6\cos x$ (e) $e^{2y+1} = e^5 + \arcsin x$

7.(6 pts.) $\lim_{x \to 0^+} (1 + \cot x)^{\frac{1}{x}} =$

(a) 0 (b) e^{-1} (c) 1 (d) ∞ (e) Does not exist

8.(6 pts.) $\lim_{x \to \infty} \frac{(\ln x)^{2.5}}{x^{0.01}} =$

(a) 0 (b) $e^{0.01}$ (c) ∞ (d) Does not exist

(e) $\ln(2.5)$

9.(6 pts.) $\displaystyle\int_0^{\frac{\sqrt{2}}{2}} \frac{dx}{\sqrt{1-x^2}} =$

(a) $\ln(\sqrt{2}-1)$ (b) $\dfrac{\pi}{4}$ (c) Diverges (d) $\dfrac{\pi}{4}-1$ (e) $\dfrac{\pi}{\sqrt{2}}$

10.(6 pts.) $\displaystyle\int_0^{\pi/2} x\cos(x)dx =$

(a) 0 (b) $1-\dfrac{\pi}{2}$ (c) $\dfrac{\sqrt{3}}{2}-\dfrac{\sqrt{2}}{2}$ (d) Diverges (e) $\dfrac{\pi}{2}-1$

11.(6 pts.) $\lim_{t \to \infty} \tanh t =$

(a) ∞ (b) $-\infty$ (c) 0 (d) Does not exist

(e) 1

12.(6 pts.) $\dfrac{x^2 + x + 2}{(x-1)(x^2+1)} =$

(a) $\dfrac{2}{(x-1)^2} - \dfrac{1}{x+1}$ (b) $\dfrac{2}{x-1} + \dfrac{3}{x^2+1}$ (c) $\dfrac{2}{x-1} + \dfrac{1}{x^2+1}$

(d) $\dfrac{2}{x-1} - \dfrac{x}{x^2+1}$ (e) $\dfrac{2}{x-1} - \dfrac{1}{x^2+1}$

13.(6 pts.) Find $\displaystyle\int_0^1 \frac{x\ dx}{x^2-1}$.

(a) 1 (b) -1 (c) 0 (d) 2 (e) Diverges

14.(6 pts.) Find $\displaystyle\sum_{n=1}^{\infty} \frac{2^{2n}}{5^{n-1}}$

(a) 5 (b) 20 (c) 4 (d) $\frac{5}{4}$ (e) $\frac{4}{5}$

15.(6 pts.) Which of the following series converge absolutely?

(1) $\displaystyle\sum_{n=0}^{\infty} \frac{\sin(2n)}{n!}$ (2) $\displaystyle\sum_{n=2}^{\infty} \frac{n}{(\ln n)^2}$ (3) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{n^3+1}$

(a) (2) and (3) converge absolutely, (1) does not

(b) (1) and (2) converge absolutely, (3) does not

(c) (1) converges absolutely, (2) and (3) do not

(d) (3) converges absolutely, (1) and (2) do not

(e) (1) and (3) converge absolutely, (2) does not

16.(6 pts.) Find $\displaystyle\lim_{n\to\infty} n \cdot \sin\left(\frac{1}{n}\right)$

(a) 0 (b) Does not exist (c) e^{-1} (d) 1

(e) ∞

17.(6 pts.) Test the following series for absolute convergence, conditional convergence or divergence:

$$(1)\ \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} \qquad (2)\ \sum_{n=0}^{\infty} \frac{(-1)^n}{(1.2)^n} \qquad (3)\ \sum_{n=0}^{\infty} \frac{(-1)^n}{n^{1.2}}$$

(a) (1) and (2) converge conditionally, (3) converges absolutely

(b) (1) converges conditionally, (2) and (3) converge absolutely

(c) (1) and (2) converge absolutely, (3) converges conditionally

(d) (1) and (3) converge absolutely, (2) converges conditionally

(e) (1) converges absolutely, (2) and (3) converge conditionally.

18.(6 pts.) Find the interval of convergence for

$$\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{\sqrt{n^2+2}}$$

Remark: $\dfrac{1}{\sqrt{n^2+2}}$ is decreasing for $n > 0$.

(a) $-1 \le x \le 1$ (b) $-1 < x < 1$ (c) $-1 \le x < 1$ (d) all x (e) $-1 < x \le 1$

19.(6 pts.) Which series below is the MacLaurin series (Taylor series centered at 0) for $x\ \sin(x^2)$?

(a) $x^2 + \dfrac{x^4}{2} + \dfrac{x^6}{3} + \cdots$ (b) $x + x^3 + x^5 + \cdots$ (c) $x^3 - \dfrac{x^7}{3!} + \dfrac{x^{11}}{5!} - \cdots$

(d) $x - x^2 + x^4 - \cdots$ (e) $x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \cdots$

20.(6 pts.) Find the order 2 MacLaurin polynomial (Taylor polynomial centered at 0) for the solution to the initial value problem

$$y' + 2y = 2x \qquad y(0) = 1$$

(a) $1 + x + x^2$ (b) $1 - \frac{1}{2!}x + \frac{1}{3!}x^2$ (c) $1 + x - \frac{1}{2!}x^2$

(d) $1 - 2x + 3x^2$ (e) $1 - 2x + \frac{2}{9}x^2$

21.(6 pts.) Which MacLaurin series (Taylor series centered at 0) represents the function $\int_0^x \cos\sqrt{t}\, dt$?

(a) $x - \frac{x^2}{4} + \frac{x^3}{3 \cdot 4!} - \cdots$

(b) $\frac{x}{2} - \frac{x^3}{2 \cdot 2!} + \frac{x^5}{3 \cdot 4!} - \cdots$

(c) $\frac{x}{2} - \frac{x^2}{2 \cdot 3!} + \frac{x^3}{3 \cdot 5!} - \cdots$

(d) The given function has no MacLaurin series

(e) $x - \frac{x^2}{2!} + \frac{x^3}{3!} - \cdots$

22.(6 pts.) The point $\left(2, \frac{7\pi}{3}\right)$ in polar coordinates corresponds to which point below in Cartesian coordinates?

(a) $(-\sqrt{3}, 1)$

(b) $(1, \sqrt{3}\,)$

(c) $(-1, \sqrt{3}\,)$

(d) $(\sqrt{3}, 1)$

(e) Since $\frac{7\pi}{3} > 2\pi$, there is no such point

23.(6 pts.) $\lim_{x \to 0} \dfrac{\ln(1+x^2) - x^2}{x^4} =$

Hint: Without MacLaurin series this may be a hard problem.

(a) Does not exist (b) ∞ (c) $-\dfrac{1}{2}$ (d) $-\infty$

(e) 0

24.(6 pts.) Find the area inside the cardioid $r = 2 + 2\cos\theta$.

cardioid.eps

(a) 6 (b) 8 (c) $3\pi + \ln 4$ (d) 6π (e) 8π

25.(6 pts.) Which graph below is the graph of the polar curve $r = 2 - 3\sin(\theta)$?

(a) graphD.eps

(b) graphE.eps

(c) graphB.eps

(d) graphC.eps

(e) graphA.eps

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