

Multiple Choice

1.(5 pts.) Let $f(x) = e^{(x^3)}$. Find $\frac{df^{-1}}{dx}(e)$.

- (a) e^3 (b) $3e$ (c) $\frac{3}{e}$ (d) $\frac{1}{2e}$ (e) $\frac{1}{3e}$

2.(5 pts.) Compute $\int_1^{e^\pi} \frac{\ln x}{x} dx$

- (a) $\frac{1}{e} - \pi$ (b) $\frac{\pi}{e^\pi}$ (c) $3\pi^2 - 2$
(d) $e\pi - e$ (e) $\frac{\pi^2}{2}$

3.(5 pts.) Find the critical point(s) of $f(x) = x \ln x$ and determine whether they are local minima, local maxima, absolute minima, absolute maxima or none of these. If a point is an absolute max/min, the answer local max/min will receive no credit.

- (a) $x = e^{-1}$, a absolute maximum
(b) There are no local maxima or local minima
(c) $x = e^{-1}$, a local minimum
(d) $x = e^{-1}$, a absolute minimum
(e) $x = e^{-1}$, a local maximum

4.(5 pts.) Find all solutions of the equation $3^{(x^2)} = 2$

- (a) $x = \frac{1}{2}\sqrt{\log_3 2}$ (b) $x = \pm\sqrt{\log_2 3}$ (c) $x = \pm\sqrt{\log_3 2}$
(d) $x = 0$ (e) $x = \frac{1}{2}\sqrt{\log_2 3}$

5.(5 pts.) Let $f(x) = \frac{x^3 e^x}{(x^2 + 1)^5}$. Find $\frac{df}{dx}$.

- (a) $\frac{-3x^2 e^x (x^2 + 1)^5 (5x^3 e^x (x^2 + 1)^4)}{(x^2 + 1)^{10}}$ (b) $\frac{3x^2 e^x + x^3 e^x}{10x(x^2 + 1)^4}$
(c) $\frac{x^3 e^x}{(x^2 + 1)^5} \left(\frac{3}{x} + 1 - 10x(x^2 + 1)^{-1} \right)$ (d) $\frac{x^3 e^x (x^2 + 1)^5 - 10x^4 e^x (x^2 + 1)^4}{(x^2 + 1)^{10}}$

(e) $\frac{x^3 e^x}{(x^2 + 1)^5} \left(\frac{(x^2 + 1)^5 - 5x^3 e^x (x^2 + 1)^4}{(x^2 + 1)^5} \right)$

6.(5 pts.) Calculate $\lim_{x \rightarrow 2^-} \frac{x^2 - 4}{x^5 - 32}$.

- (a) $\frac{1}{20}$ (b) $\frac{1}{40}$ (c) $\frac{1}{10}$ (d) $\frac{1}{5}$ (e) $\frac{2}{5}$

7.(5 pts.) Calculate $\lim_{x \rightarrow 2} x^x$.

- (a) 4 (b) 3 (c) 2
(d) Limit does not exist. (e) 1

8.(5 pts.) Calculate $\int_0^1 \frac{x}{\sqrt{1-x^4}} dx$.

- (a) $\operatorname{arcsec}(1)$ (b) $\frac{1}{2} \arcsin(1)$ (c) $\frac{1}{2} \operatorname{arcsec}(1)$ (d) $\frac{1}{2} \arctan(1)$ (e) $2 \arctan(1)$

9.(5 pts.) Calculate $\frac{d}{dx} \arctan(\ln x)$.

- (a) $\frac{1}{\ln x(1+x^2)}$ (b) $\frac{1}{x(1+(\ln x)^2)}$
(c) $\frac{\ln x}{(1+x^2)}$ (d) $\frac{x}{(1+(\ln x)^2)}$
(e) $\frac{1}{(1+(\ln x)^2)}$

10.(5 pts.) Calculate $\int_2^3 \frac{dx}{(x+1)\sqrt{x^2+2x}}$.

- (a) $\operatorname{arcsec}(3) - \operatorname{arcsec}(2)$ (b) $\arctan(3) - \arctan(1)$
(c) $\arctan(2) - \arctan(1)$ (d) $\arcsin(\frac{1}{3}) - \arcsin(\frac{1}{2})$
(e) $\operatorname{arcsec}(4) - \operatorname{arcsec}(3)$

Partial Credit

11.(10 pts.) Consider the function $f(x) = \sqrt{2x^4 + x^2}$.

- a) Show that f is one to one on the domain $(0, \infty)$.

- b) Find the slope of the tangent line to the graph of the inverse function f^{-1} at the point $f^{-1}(6) = 2$.

12.(10 pts.) Find the derivative of the function

$$f(x) = \sqrt[x]{x} = x^{\frac{1}{x}} \quad .$$

13.(10 pts.) The quantity of a radioactive substance decreases from 100% to 80% in three hours. Compute the half-life (the time until you have 50% of your sample left) as a quotient of logs.

14.(10 pts.) Determine $\lim_{x \rightarrow \infty} \sqrt{e^x + x} - \sqrt{e^x + 1}$.

Hint: Rewrite the expression using algebra and then use what you know about rates of growth.

15.(10 pts.) Express $\sec(\arctan(x))$ as an algebraic function of x .

Name: _____

Instructor: Bullwinkle

Exam I
February 15, 2000

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for one hour.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 11 pages of the test.

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

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|-----|-----|-----|-----|-----|-----|
| 1. | (a) | (b) | (c) | (d) | (●) |
| 2. | (a) | (b) | (c) | (d) | (●) |
| 3. | (a) | (b) | (c) | (●) | (e) |
| 4. | (a) | (b) | (●) | (d) | (e) |
| 5. | (a) | (b) | (●) | (d) | (e) |
| 6. | (●) | (b) | (c) | (d) | (e) |
| 7. | (●) | (b) | (c) | (d) | (e) |
| 8. | (a) | (●) | (c) | (d) | (e) |
| 9. | (a) | (●) | (c) | (d) | (e) |
| 10. | (a) | (b) | (c) | (d) | (●) |

DO NOT WRITE IN THIS BOX!

Total multiple choice: _____

11. _____

12. _____

13. _____

14. _____

15. _____

Total: _____